

PRIME FACTORIZATION AND DOMINATION IN THE HIERARCHICAL PRODUCT OF GRAPHS

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Abstract

In 2009, Barrière, Dalfó, Fiol, and Mitjana introduced the generalized hierarchical product of graphs. This operation is a generalization of the Cartesian product of graphs. It is known that every connected graph has a unique prime factor decomposition with respect to the Cartesian product. We generalize this result to show that connected graphs indeed have a unique prime factor decomposition with respect to the generalized hierarchical product. We also give preliminary results on the domination number of generalized hierarchical products.

Keywords: generalized hierarchical product, Cartesian product, prime factor decomposition.

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