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EDGE-TRANSITIVITY OF CAYLEY GRAPHS GENERATED BY TRANSPOSITIONS

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Abstract

Let S be a set of transpositions generating the symmetric group S_n ($n \geq 5$). The transposition graph of S is defined to be the graph with vertex set $\{1, \ldots, n\}$, and with vertices i and j being adjacent in $T(S)$ whenever $(i, j) \in S$. In the present note, it is proved that two transposition graphs are isomorphic if and only if the corresponding two Cayley graphs are isomorphic. It is also proved that the transposition graph $T(S)$ is edge-transitive if and only if the Cayley graph $\text{Cay}(S_n, S)$ is edge-transitive.

Keywords: Cayley graphs, transpositions, automorphisms of graphs, edge-transitive graphs, line graphs, Whitney's isomorphism theorem.

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