

THE CROSSING NUMBER OF THE HEXAGONAL GRAPH $H_{3,n}$

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Abstract

In [C. Thomassen, *Tilings of the torus and the Klein bottle and vertex-transitive graphs on a fixed surface*, Trans. Amer. Math. Soc. 323 (1991) 605–635], Thomassen described completely all (except finitely many) regular tilings of the torus S_1 and the Klein bottle N_2 into (3,6)-tilings, (4,4)-tilings and (6,3)-tilings. Many authors made great efforts to investigate the crossing number (in the plane) of the Cartesian product of an m -cycle and an n -cycle, which is a special (4,4)-tiling. For other tilings, there are quite rare results concerning on their crossing numbers. This motivates us in the paper to determine the crossing number of a hexagonal graph $H_{3,n}$, which is a special kind of (3,6)-tilings.

Keywords: hexagonal graph, Cartesian product, crossing number, drawing.

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