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ON THE TOTAL ROMAN DOMINATION IN TREES

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Abstract

A *total Roman dominating function* on a graph G is a function $f : V(G) \to \{0, 1, 2\}$ satisfying the following conditions: (i) every vertex u for which $f(u) = 0$ is adjacent to at least one vertex v for which $f(v) = 2$ and (ii) the subgraph of G induced by the set of all vertices of positive weight has no isolated vertex. The weight of a total Roman dominating function f is the value $f(V(G)) = \Sigma_{u \in V(G)} f(u)$. The *total Roman domination number* $\gamma_{tR}(G)$ is the minimum weight of a total Roman dominating function of G. Ahangar *et al.* in [H.A. Ahangar, M.A. Henning, V. Samodivkin and I.G. Yero, *Total Roman domination in graphs*, Appl. Anal. Discrete Math. 10 (2016) 501–517] recently showed that for any graph G without isolated vertices, $2\gamma(G) \leq \gamma_{tR}(G) \leq 3\gamma(G)$, where $\gamma(G)$ is the domination number of G, and they raised the problem of characterizing the graphs G achieving these upper and lower bounds. In this paper, we provide a constructive characterization of these trees.

Keywords: total Roman dominating function, total Roman domination number, trees.

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References

[1] H. Abdollahzadeh Ahangar, J. Amjadi, S.M. Sheikholeslami and M. Soroudi, *Bounds on the total Roman domination numbers*, Ars Combin., to appear.

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[2] H. Abdollahzadeh Ahangar, A. Bahremandpour, S.M. Sheikholeslami, N.D. Soner, Z. Tahmasbzadehbaee and L. Volkmann, *Maximal Roman domination numbers in graphs*, Util. Math. **103** (2017) 245–258.

[3] H. Abdollahzadeh Ahangar, M.A. Henning, V. Samodivkin and I.G. Yero, *Total Roman domination in graphs*, Appl. Anal. Discrete Math. **10** (2016) 501–517. doi:10.2298/AADM160802017A

[4] H. Abdollahzadeh Ahangar, T.W. Haynes and J.C. Valenzuela-Tripodoro, *Mixed Roman domination in graphs*, Bull. Malays. Math. Sci. Soc. **40** (2017) 1443–1454. doi:10.1007/s40840-015-0141-1

[5] J. Amjadi, S. Nazari-Moghaddam and S.M. Sheikholeslami, *Global total Roman domination in graphs*, Discrete Math. Algorithms Appl. **09** (2017) ID: 1750050. doi:10.1142/S1793830917500501

[6] J. Amjadi, S. Nazari-Moghaddam, S.M. Sheikholeslami and L. Volkmann, *Total Roman domination number of trees*, Australas. J. Combin. **69** (2017) 271–285.

[7] J. Amjadi, S.M. Sheikholeslami and M. Soroudi, *Nordhaus-Gaddum bounds for total Roman domination*, J. Comb. Optim. **35** (2018) 126–133. doi:10.1007/s10878-017-0158-5

[8] R.A. Beeler, T.W. Haynes and S.T. Hedetniemi, *Double Roman domination*, Discrete Appl. Math. **211** (2016) 23–29. doi:10.1016/j.dam.2016.03.017

[9] M. Chellali, T.W. Haynes, S.T. Hedetniemi and A.A. McRae, *Roman {2}-domination*, Discrete Appl. Math. **204** (2016) 22–28. doi:10.1016/j.dam.2015.11.013

[10] E.J. Cockayne, P.A. Dreyer, S.M. Hedetniemi and S.T. Hedetniemi, *Roman domination in graphs*, Discrete Math. **278** (2004) 11–22. doi:10.1016/j.disc.2003.06.004

[11] M.R. Garey and D.S. Johnson, Computers and Intractability: a Guide to the Theory of NP-Completeness (W.H. Freeman and Co., San Francisco, Calif., 1979).

[12] T.W. Haynes, S.T. Hedetniemi and P.J. Slater, Fundamentals of Domination in Graphs (Marcel Dekker Inc., New York, 1998).

[13] T.W. Haynes, S.T. Hedetniemi and P.J. Slater, Domination in Graphs: Advanced Topics (Marcel Dekker Inc., New York, 1998).

[14] M.A. Henning and S.T. Hedetniemi, *Defending the Roman Empire—A new strategy*, Discrete Math. **266** (2003) 239–251. doi:10.1016/S0012-365X(02)00811-7

[15] L.L. Kelleher and M.B. Cozzens, *Dominating sets in social network graphs*, Math. Social Sci. **16** (1988) 267–279. doi:10.1016/0165-4896(88)90041-8

[16] C.-H. Liu and G.J. Chang, *Roman domination on strongly chordal graphs*, J. Comb. Optim. **26** (2013) 608–619. doi:10.1007/s10878-012-9482-y

[17] C.S. ReVelle and K.E. Rosing, *Defendens imperium Romanum: a classical problem in military strategy*, Amer. Math. Monthly **107** (2000) 585–594. doi:10.1080/00029890.2000.12005243

[18] I. Stewart, *Defend the Roman Empire!*, Sci. Amer. **281** (1999) 136–138. doi:10.1038/scientificamerican1299-136

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