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ANTIPODAL EDGE-COLORINGS OF HYPERCUBES

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Abstract

Two vertices of the k-dimensional hypercube Q_k are *antipodal* if they differ in every coordinate. Edges uv and xy are *antipodal* if u is antipodal to x and v is antipodal to y. An *antipodal edge-coloring* of Q_k is a 2-edge-coloring such that antipodal edges always have different colors. Norine conjectured that for $k \geq 2$, in every antipodal edge-coloring of Q_k some two antipodal vertices are connected by a monochromatic path. Feder and Subi proved this for $k \leq 5$. We prove it for $k \leq 6$.

Keywords: antipodal edge-coloring, hypercube, monochromatic geodesic.

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