

HOMOMORPHIC PREIMAGES OF GEOMETRIC PATHS

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Abstract

A graph G is a homomorphic preimage of another graph H , or equivalently G is H -colorable, if there exists a graph homomorphism $f : G \rightarrow H$. A geometric graph \overline{G} is a simple graph G together with a straight line drawing of G in the plane with the vertices in general position. A geometric homomorphism (respectively, isomorphism) $\overline{G} \rightarrow \overline{H}$ is a graph homomorphism (respectively, isomorphism) that preserves edge crossings (respectively, and non-crossings). The homomorphism poset \mathcal{G} of a graph G is the set of isomorphism classes of geometric realizations of G partially ordered by the existence of injective geometric homomorphisms. A geometric graph \overline{G} is \mathcal{H} -colorable if $\overline{G} \rightarrow \overline{H}$ for some $\overline{H} \in \mathcal{H}$. In this paper, we provide necessary and sufficient conditions for \overline{G} to be \mathcal{P}_n -colorable for $n \geq 2$. Along the way, we also provide necessary and sufficient conditions for \overline{G} to be $\mathcal{K}_{2,3}$ -colorable.

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