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SOME RESULTS ON THE INDEPENDENCE POLYNOMIAL OF UNICYCLIC GRAPHS

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Abstract

Let G be a simple graph on n vertices. An independent set in a graph is a set of pairwise non-adjacent vertices. The independence polynomial of G is the polynomial $I(G,x)=\sum_{k=0}^{n} s(G,k)x^k$, where $s(G,k)$ is the number of independent sets of G with size k and $s(G,0)=1$. A unicyclic graph is a graph containing exactly one cycle. Let C_n be the cycle on n vertices. In this paper we study the independence polynomial of unicyclic graphs. We show that among all connected unicyclic graphs G on n vertices (except two of them), $I(G,t) > I(C_n,t)$ for sufficiently large t. Finally for every $n \geq 3$ we find all connected graphs H such that $I(H,x)=I(C_n,x)$.

Keywords: independence polynomial, independent set, unicyclic graphs.

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