THE STEINER WIENER INDEX OF A GRAPH

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Abstract

The Wiener index \( W(G) \) of a connected graph \( G \), introduced by Wiener in 1947, is defined as \( W(G) = \sum_{u,v \in V(G)} d(u,v) \) where \( d_G(u,v) \) is the distance between vertices \( u \) and \( v \) of \( G \). The Steiner distance in a graph, introduced by Chartrand et al. in 1989, is a natural generalization of the concept of classical graph distance. For a connected graph \( G \) of order at least 2 and \( S \subseteq V(G) \), the Steiner distance \( d(S) \) of the vertices of \( S \) is the minimum size of a connected subgraph whose vertex set is \( S \). We now introduce the concept of the Steiner Wiener index of a graph. The Steiner \( k \)-Wiener index \( SW_k(G) \) of \( G \) is defined by \( \text{SW}_k(G) = \sum_{S \subseteq V(G)} |S|^k d(S) \). Expressions for \( \text{SW}_k \) for some special graphs are obtained. We also give sharp upper and lower bounds of \( \text{SW}_k \) of a connected graph, and establish some of its properties in

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the case of trees. An application in chemistry of the Steiner Wiener index is reported in our another paper.

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**References**


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