CHARACTERIZATIONS OF GRAPHS HAVING LARGE PROPER CONNECTION NUMBERS

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Abstract

Let $G$ be an edge-colored connected graph. A path $P$ is a proper path in $G$ if no two adjacent edges of $P$ are colored the same. If $P$ is a proper $u-v$ path of length $d(u,v)$, then $P$ is a proper $u-v$ geodesic. An edge coloring $c$ is a proper-path coloring of a connected graph $G$ if every pair $u,v$ of distinct vertices of $G$ are connected by a proper $u-v$ path in $G$, and $c$ is a strong proper-path coloring if every two vertices $u$ and $v$ are connected by a proper $u-v$ geodesic in $G$. The minimum number of colors required for a proper-path coloring or strong proper-path coloring of $G$ is called the proper connection number $pc(G)$ or strong proper connection number $spc(G)$ of $G$, respectively. If $G$ is a nontrivial connected graph of size $m$, then $pc(G) \leq spc(G) \leq m$ and $pc(G) = m$ or $spc(G) = m$ if and only if $G$ is the star of size $m$. In this paper, we determine all connected graphs $G$ of size $m$ for which $pc(G)$ or $spc(G)$ is $m-1$, $m-2$ or $m-3$.

Keywords: edge coloring, proper-path coloring, strong proper-path coloring.

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References

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