

## CHARACTERIZATIONS OF GRAPHS HAVING LARGE PROPER CONNECTION NUMBERS

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### Abstract

Let  $G$  be an edge-colored connected graph. A path  $P$  is a proper path in  $G$  if no two adjacent edges of  $P$  are colored the same. If  $P$  is a proper  $u - v$  path of length  $d(u, v)$ , then  $P$  is a proper  $u - v$  geodesic. An edge coloring  $c$  is a proper-path coloring of a connected graph  $G$  if every pair  $u, v$  of distinct vertices of  $G$  are connected by a proper  $u - v$  path in  $G$ , and  $c$  is a strong proper-path coloring if every two vertices  $u$  and  $v$  are connected by a proper  $u - v$  geodesic in  $G$ . The minimum number of colors required for a proper-path coloring or strong proper-path coloring of  $G$  is called the proper connection number  $pc(G)$  or strong proper connection number  $spc(G)$  of  $G$ , respectively. If  $G$  is a nontrivial connected graph of size  $m$ , then  $pc(G) \leq spc(G) \leq m$  and  $pc(G) = m$  or  $spc(G) = m$  if and only if  $G$  is the star of size  $m$ . In this paper, we determine all connected graphs  $G$  of size  $m$  for which  $pc(G)$  or  $spc(G)$  is  $m - 1, m - 2$  or  $m - 3$ .

**Keywords:** edge coloring, proper-path coloring, strong proper-path coloring.

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