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DECOMPOSITION OF THE PRODUCT OF CYCLES BASED ON DEGREE PARTITION

Y.M. Borse and S.R. Shaikh

Department of Mathematics
Savitribai Phule Pune University
Pune-411007, India

e-mail: ymborse11@gmail.com
shazia_31082@yahoo.co.in

Abstract

The Cartesian product of n cycles is a $2n$-regular, $2n$-connected and bipancyclic graph. Let G be the Cartesian product of n even cycles and let $2n = n_1 + n_2 + \cdots + n_k$ with $k \geq 2$ and $n_i \geq 2$ for each i. We prove that if $k = 2$, then G can be decomposed into two spanning subgraphs G_1 and G_2 such that each G_i is n_i-regular, n_i-connected, and bipancyclic or nearly bipancyclic. For $k > 2$, we establish that if all n_i in the partition of $2n$ are even, then G can be decomposed into k spanning subgraphs $G_1, G_2, \ldots, G_k$ such that each G_i is n_i-regular and n_i-connected. These results are analogous to the corresponding results for hypercubes.

Keywords: hypercube, Cartesian product, n-connected, regular, bipancyclic, spanning subgraph.

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