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ORIENTED CHROMATIC NUMBER OF CARTESIAN PRODUCTS AND STRONG PRODUCTS OF PATHS

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Abstract

An oriented coloring of an oriented graph G is a homomorphism from G to H such that H is without selfloops and arcs in opposite directions. We shall say that H is a *coloring graph*. In this paper, we focus on oriented colorings of Cartesian products of two paths, called grids, and strong products of two paths, called strong-grids. We show that there exists a coloring graph with nine vertices that can be used to color every orientation of grids with five columns. We also show that there exists a strong-grid with two columns and its orientation which requires 11 colors for oriented coloring. Moreover, we show that every orientation of every strong-grid with three columns can be colored by 19 colors and that every orientation of every strong-grid with four columns can be colored by 43 colors. The above statements were proved with the help of computer programs.

Keywords: graph, oriented coloring, grid.

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