

## ON THE HAMILTONIAN NUMBER OF A PLANE GRAPH

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### Abstract

The Hamiltonian number of a connected graph is the minimum of the lengths of the closed spanning walks in the graph. In 1968, Grinberg published a necessary condition for the existence of a Hamiltonian cycle in a plane graph, formulated in terms of the degrees of its faces. We show how Grinberg's theorem can be adapted to provide a lower bound on the Hamiltonian number of a plane graph.

**Keywords:** Hamiltonian cycle, Hamiltonian walk, Hamiltonian number, Hamiltonian spectrum, Grinberg's theorem, planar graph.

**2010 Mathematics Subject Classification:** 05C10.

### REFERENCES

- [1] M. Araya and G. Wiener, *On cubic planar hypohamiltonian and hypotraceable graphs*, *Electron. J. Combin.* **18** (2011) #P85.
- [2] T. Asano, T. Nishizeki and T. Watanabe, *An upper bound on the length of a Hamiltonian walk of a maximal planar graph*, *J. Graph Theory* **4** (1980) 315–336.  
doi:10.1002/jgt.3190040310
- [3] J.-C. Bermond, *On Hamiltonian walks*, in: *Proceedings of the Fifth British Combinatorial Conference*, *Util. Math.*, Winnipeg, Man. (1975) 41–51.
- [4] J.A. Bondy and U.S.R. Murty, *Graph Theory* (Springer, New York, 2008).
- [5] G.J. Chang, T.-D. Chang and L.-D. Tong, *Hamiltonian numbers of Möbius double loop networks*, *J. Comb. Optim.* **23** (2012) 462–470.  
doi:10.1007/s10878-010-9360-4
- [6] T.-D. Chang and L.-D. Tong, *The Hamiltonian numbers in digraphs*, *J. Combin. Optim.* **25** (2013) 694–701.  
doi:10.1007/s10878-012-9512-9

- [7] G. Chartrand and P. Zhang, *A First Course in Graph Theory* (Dover Publications, Incorporated, 2012).
- [8] G. Chartrand, T. Thomas, P. Zhang and V. Saenpholphat, *A new look at Hamiltonian walks*, *Bull. Inst. Combin. Appl.* **42** (2004) 37–52.
- [9] S.E. Goodman and S.T. Hedetniemi, *On Hamiltonian walks in graphs*, *SIAM J. Comput.* **3** (1974) 214–221.  
doi:10.1137/0203017
- [10] S.E. Goodman, S.T. Hedetniemi and P.J. Slater, *Advances on the Hamiltonian completion problem*, *J. Association Computing Machinery* **22** (1975) 352–360.  
doi:10.1145/321892.321897
- [11] E. Grinberg, *Plane homogeneous graphs of degree three without Hamiltonian circuits*, *Latvian Math. Yearbook* **4**, Izdat. “Zinatne”, Riga (1968) 51–58, in Russian.
- [12] D. Král, L.-D. Tong and X. Zhu, *Upper Hamiltonian numbers and Hamiltonian spectra of graphs*, *Australas. J. Combin.* **35** (2006) 329–340.
- [13] D. Liu, *Hamiltonian spectra of trees*, *Ars Combin.* **99** (2011) 415–419.
- [14] T. Nishizeki, T. Asano and T. Watanabe, *An approximation algorithm for the Hamiltonian walk problem on maximal planar graphs*, *Discrete Appl. Math.* **5** (1983) 211–222.  
doi:10.1016/0166-218X(83)90042-2
- [15] F. Okamoto, P. Zhang and V. Saenpholphat, *The upper traceable number of a graph*, *Czechoslovak Math. J.* **58** (2008) 271–287.  
doi:10.1007/s10587-008-0016-9
- [16] N. Punnim, V. Saenpholphat and S. Thaitae, *Almost Hamiltonian cubic graphs*, *Internat. J. Comput. Sci. Inform. Security* **7** (2007) 83–86.
- [17] N. Punnim and S. Thaitae, *The Hamiltonian number of some classes of cubic graphs*, *East-West J. Math.* **12** (2010) 17–26.
- [18] V. Saenpholphat, F. Okamoto and P. Zhang, *Measures of traceability in graphs*, *Math. Bohem.* **131** (2006) 63–83.
- [19] S. Thaitae and N. Punnim, *The Hamiltonian number of cubic graphs*, in: *Computational Geometry and Graph Theory, Lecture Notes in Comput. Sci.* **4535**, H. Ito, M. Kano, N. Katoh and Y. Uno (Ed(s)), (Springer, Berlin, Heidelberg, 2008) 213–223.  
doi:10.1007/978-3-540-89550-3\_23
- [20] P. Vacek, *On open Hamiltonian walks in graphs*, *Arch. Math. (Brno)* **27A** (1991) 105–111.
- [21] P. Vacek, *Bounds of lengths of open Hamiltonian walks*, *Arch. Math. (Brno)* **28** (1992) 11–16.
- [22] G. Wiener and M. Araya, *On planar hypohamiltonian graphs*, *J. Graph Theory* **67** (2011) 55–68.  
doi:10.1002/jgt.20513

Received 12 July 2016  
Revised 10 May 2017  
Accepted 3 July 2017