

## BOUNDS FOR THE $b$ -CHROMATIC NUMBER OF SUBGRAPHS AND EDGE-DELETED SUBGRAPHS

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### Abstract

A  $b$ -coloring of a graph  $G$  with  $k$  colors is a proper coloring of  $G$  using  $k$  colors in which each color class contains a color dominating vertex, that is, a vertex which has a neighbor in each of the other color classes. The largest positive integer  $k$  for which  $G$  has a  $b$ -coloring using  $k$  colors is the  $b$ -chromatic number  $b(G)$  of  $G$ . In this paper, we obtain bounds for the  $b$ -chromatic number of induced subgraphs in terms of the  $b$ -chromatic number of the original graph. This turns out to be a generalization of the result due to R. Balakrishnan *et al.* [*Bounds for the  $b$ -chromatic number of  $G - v$* , *Discrete Appl. Math.* 161 (2013) 1173–1179]. Also we show that for any connected graph  $G$  and any  $e \in E(G)$ ,  $b(G - e) \leq b(G) + \lceil \frac{n}{2} \rceil - 2$ . Further, we determine all graphs which attain the upper bound. Finally, we conclude by finding bound for the  $b$ -chromatic number of any subgraph.

**Keywords:**  $b$ -coloring,  $b$ -chromatic number.

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