

## GRAPHS WITH LARGE GENERALIZED (EDGE-)CONNECTIVITY

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### Abstract

The generalized  $k$ -connectivity  $\kappa_k(G)$  of a graph  $G$ , introduced by Hager in 1985, is a nice generalization of the classical connectivity. Recently, as a natural counterpart, we proposed the concept of generalized  $k$ -edge-connectivity  $\lambda_k(G)$ . In this paper, graphs of order  $n$  such that  $\kappa_k(G) = n - \frac{k}{2} - 1$  and  $\lambda_k(G) = n - \frac{k}{2} - 1$  for even  $k$  are characterized.

**Keywords:** (edge-)connectivity, Steiner tree, internally disjoint trees, edge-disjoint trees, packing, generalized (edge-)connectivity.

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