

## FORBIDDEN SUBGRAPHS FOR HAMILTONICITY OF 1-TOUGH GRAPHS

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### Abstract

A graph  $G$  is said to be 1-tough if for every vertex cut  $S$  of  $G$ , the number of components of  $G - S$  does not exceed  $|S|$ . Being 1-tough is an obvious necessary condition for a graph to be hamiltonian, but it is not sufficient in general. We study the problem of characterizing all graphs  $H$  such that every 1-tough  $H$ -free graph is hamiltonian. We almost obtain a complete solution to this problem, leaving  $H = K_1 \cup P_4$  as the only open case.

**Keywords:** forbidden subgraph, 1-tough graph,  $H$ -free graph, hamiltonian graph.

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