

## THE DYNAMICS OF THE FOREST GRAPH OPERATOR

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### Abstract

In 1966, Cummins introduced the “tree graph”: the tree graph  $\mathbf{T}(G)$  of a graph  $G$  (possibly infinite) has all its spanning trees as vertices, and distinct such trees correspond to adjacent vertices if they differ in just one edge, i.e., two spanning trees  $T_1$  and  $T_2$  are adjacent if  $T_2 = T_1 - e + f$  for some edges  $e \in T_1$  and  $f \notin T_1$ . The tree graph of a connected graph need not be connected. To obviate this difficulty we define the “forest graph”: let  $G$  be a labeled graph of order  $\alpha$ , finite or infinite, and let  $\mathfrak{N}(G)$  be the set of all labeled maximal forests of  $G$ . The forest graph of  $G$ , denoted by  $\mathbf{F}(G)$ , is the graph with vertex set  $\mathfrak{N}(G)$  in which two maximal forests  $F_1, F_2$  of  $G$  form an edge if and only if they differ exactly by one edge, i.e.,  $F_2 = F_1 - e + f$  for some edges  $e \in F_1$  and  $f \notin F_1$ .

Using the theory of cardinal numbers, Zorn's lemma, transfinite induction, the axiom of choice and the well-ordering principle, we determine the  $\mathbf{F}$ -convergence,  $\mathbf{F}$ -divergence,  $\mathbf{F}$ -depth and  $\mathbf{F}$ -stability of any graph  $G$ . In particular it is shown that a graph  $G$  (finite or infinite) is  $\mathbf{F}$ -convergent if and only if  $G$  has at most one cycle of length 3. The  $\mathbf{F}$ -stable graphs are precisely  $K_3$  and  $K_1$ . The  $\mathbf{F}$ -depth of any graph  $G$  different from  $K_3$  and  $K_1$  is finite. We also determine various parameters of  $\mathbf{F}(G)$  for an infinite graph  $G$ , including the number, order, size, and degree of its components.

**Keywords:** forest graph operator, graph dynamics.

**2010 Mathematics Subject Classification:** Primary 05C76; Secondary 05C05, 05C63.

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Received 31 July 2015  
Revised 6 January 2016  
Accepted 6 January 2016