AN EXTENSION OF KOTZIG’S THEOREM

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Abstract

In 1955, Kotzig proved that every 3-connected planar graph has an edge with the degree sum of its end vertices at most 13, which is tight. An edge uv is of type \((i, j)\) if \(d(u) \leq i\) and \(d(v) \leq j\). Borodin (1991) proved that every normal plane map contains an edge of one of the types \((3, 10)\), \((4, 7)\), or \((5, 6)\), which is tight. Cole, Kowalik, and Škrekovski (2007) deduced from this result by Borodin that Kotzig’s bound of 13 is valid for all planar graphs with minimum degree \(\delta\) at least 2 in which every \(d\)-vertex, \(d \geq 12\), has at most \(d - 11\) neighbors of degree 2.

We give a common extension of the three above results by proving for any integer \(t \geq 1\) that every plane graph with \(\delta \geq 2\) and no \(d\)-vertex, \(d \geq 11 + t\), having more than \(d - 11\) neighbors of degree 2 has an edge of one of the following types: \((2, 10 + t)\), \((3, 10)\), \((4, 7)\), or \((5, 6)\), where all parameters are tight.

Keywords: plane graph, normal plane map, structural property, weight.

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References


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