

REGULARITY AND PLANARITY OF TOKEN GRAPHS

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Abstract

Let $G = (V, E)$ be a graph of order n and let $1 \leq k < n$ be an integer. The k -token graph of G is the graph whose vertices are all the k -subsets of V , two of which are adjacent whenever their symmetric difference is a pair of adjacent vertices in G . In this paper we characterize precisely, for each value of k , which graphs have a regular k -token graph and which connected graphs have a planar k -token graph.

Keywords: token graph, Johnson graph, regularity, planarity.

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