

A DECOMPOSITION OF GALLAI MULTIGRAPHS

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Abstract

An edge-colored cycle is *rainbow* if its edges are colored with distinct colors. A *Gallai* (multi)graph is a simple, complete, edge-colored (multi)graph lacking rainbow triangles. As has been previously shown for Gallai graphs, we show that Gallai multigraphs admit a simple iterative construction. We then use this structure to prove Ramsey-type results within Gallai colorings. Moreover, we show that Gallai multigraphs give rise to a surprising and highly structured decomposition into directed trees.

Keywords: edge coloring, Gallai multigraph.

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