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RANK NUMBERS FOR BENT LADDERS

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Abstract

A ranking on a graph is an assignment of positive integers to its vertices such that any path between two vertices with the same label contains a vertex with a larger label. The rank number of a graph is the fewest number of labels that can be used in a ranking. The rank number of a graph is known for many families, including the ladder graph $P_2 \times P_n$. We consider how "bending" a ladder affects the rank number. We prove that in certain cases the rank number does not change, and in others the rank number differs by only 1. We investigate the rank number of a ladder with an arbitrary number of bends.

Keywords: graph colorings, rankings of graphs, rank number, Cartesian product of graphs, ladder graph, bent ladder graph.

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