

## ON THE UNIQUENESS OF $D$ -VERTEX MAGIC CONSTANT

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### Abstract

Let  $G = (V, E)$  be a graph of order  $n$  and let  $D \subseteq \{0, 1, 2, 3, \dots\}$ . For  $v \in V$ , let  $N_D(v) = \{u \in V : d(u, v) \in D\}$ . The graph  $G$  is said to be  $D$ -vertex magic if there exists a bijection  $f : V(G) \rightarrow \{1, 2, \dots, n\}$  such that for all  $v \in V$ ,  $\sum_{u \in N_D(v)} f(u)$  is a constant, called  $D$ -vertex magic constant. O'Neal and Slater have proved the uniqueness of the  $D$ -vertex magic constant by showing that it can be determined by the  $D$ -neighborhood fractional domination number of the graph. In this paper we give a simple and elegant proof of this result. Using this result, we investigate the existence of distance magic labelings of complete  $r$ -partite graphs where  $r \geq 4$ .

**Keywords:** distance magic graph,  $D$ -vertex magic graph, magic constant, dominating function, fractional domination number.

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