

HEAVY SUBGRAPH PAIRS FOR TRACEABILITY OF BLOCK-CHAINS

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Abstract

A graph is called traceable if it contains a Hamilton path, i.e., a path containing all its vertices. Let G be a graph on n vertices. We say that an induced subgraph of G is o_{-1} -heavy if it contains two nonadjacent vertices which satisfy an Ore-type degree condition for traceability, i.e., with degree sum at least $n-1$ in G . A block-chain is a graph whose block graph is a path, i.e., it is either a P_1 , P_2 , or a 2-connected graph, or a graph with at least one cut vertex and exactly two end-blocks. Obviously, every traceable graph is a block-chain, but the reverse does not hold. In this paper we characterize all the pairs of connected o_{-1} -heavy graphs that guarantee traceability of block-chains. Our main result is a common extension of earlier work on degree sum conditions, forbidden subgraph conditions and heavy subgraph conditions for traceability.

Keywords: o_{-1} -heavy subgraph, block-chain traceable graph, Ore-type condition, forbidden subgraph.

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