

FAMILIES OF TRIPLES WITH HIGH MINIMUM DEGREE ARE HAMILTONIAN

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Abstract

In this paper we show that every family of triples, that is, a 3-uniform hypergraph, with minimum degree at least $(\frac{5-\sqrt{5}}{3} + \gamma)\binom{n-1}{2}$ contains a tight Hamiltonian cycle.

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