

THE IRREGULARITY OF GRAPHS UNDER GRAPH OPERATIONS

HOSAM ABDO AND DARKO DIMITROV

*Institut für Informatik, Freie Universität Berlin
Takustraße 9, D-14195 Berlin, Germany*

e-mail: abdo,darko@mi.fu-berlin.de

Abstract

The *irregularity* of a simple undirected graph G was defined by Albertson [5] as $\text{irr}(G) = \sum_{uv \in E(G)} |d_G(u) - d_G(v)|$, where $d_G(u)$ denotes the degree of a vertex $u \in V(G)$. In this paper we consider the irregularity of graphs under several graph operations including join, Cartesian product, direct product, strong product, corona product, lexicographic product, disjunction and symmetric difference. We give exact expressions or (sharp) upper bounds on the irregularity of graphs under the above mentioned operations.

Keywords: irregularity of graphs, total irregularity of graphs, graph operations, Zagreb indices.

2010 Mathematics Subject Classification: 05C30, 05C76, 05C90.

REFERENCES

- [1] H. Abdo and D. Dimitrov, *Total irregularity of a graph*, (2012) a manuscript. arxiv.org/abs/1207.5267
- [2] Y. Alavi, A. Boals, G. Chartrand, P. Erdős and O.R. Oellermann, *k-path irregular graphs*, Congr. Numer. **65** (1988) 201–210.
- [3] Y. Alavi, G. Chartrand, F.R.K. Chung, P. Erdős, R.L. Graham and O.R. Oellermann, *Highly irregular graphs*, J. Graph Theory **11** (1987) 235–249. doi:10.1002/jgt.3190110214
- [4] Y. Alavi, J. Liu and J. Wang, *Highly irregular digraphs*, Discrete Math. **111** (1993) 3–10. doi:10.1016/0012-365X(93)90134-F
- [5] M.O. Albertson, *The irregularity of a graph*, Ars Combin. **46** (1997) 219–225.
- [6] M.O. Albertson and D. Berman, *Ramsey graphs without repeated degrees*, Congr. Numer. **83** (1991) 91–96.

- [7] F.K. Bell, *A note on the irregularity of graphs*, Linear Algebra Appl. **161** (1992) 45–54.
doi:10.1016/0024-3795(92)90004-T
- [8] F.K. Bell, *On the maximal index of connected graphs*, Linear Algebra Appl. **144** (1991) 135–151.
doi:10.1016/0024-3795(91)90067-7
- [9] G. Chartrand, P. Erdős and O.R. Oellermann, *How to define an irregular graph*, College Math. J. **19** (1988) 36–42.
doi:10.2307/2686701
- [10] G. Chartrand, K.S. Holbert, O.R. Oellermann and H.C. Swart, *F-degrees in graphs*, Ars Combin. **24** (1987) 133–148.
- [11] G. Chen, P. Erdős, C. Rousseau and R. Schelp, *Ramsey problems involving degrees in edge-colored complete graphs of vertices belonging to monochromatic subgraphs*, European J. Combin. **14** (1993) 183–189.
doi:10.1006/eujc.1993.1023
- [12] L. Collatz and U. Sinogowitz, *Spektren endlicher Graphen*, Abh. Math. Semin. Univ. Hamburg **21** (1957) 63–77.
doi:10.1007/BF02941924
- [13] D. Cvetković and P. Rowlinson, *On connected graphs with maximal index*, Publications de l’Institut Mathématique Beograd **44** (1988) 29–34.
- [14] K.C. Das and I. Gutman, *Some properties of the second Zagreb index*, MATCH Commun. Math. Comput. Chem. **52** (2004) 103–112.
- [15] T. Došlić, B. Furtula, A. Graovac, I. Gutman, S. Moradi and Z. Yarahmadi, *On vertex degree based molecular structure descriptors*, MATCH Commun. Math. Comput. Chem. **66** (2011) 613–626.
- [16] G.H. Fath-Tabar, *Old and new Zagreb indices of graphs*, MATCH Commun. Math. Comput. Chem. **65** (2011) 79–84.
- [17] I. Gutman and K.C. Das, *The first Zagreb index 30 years after*, MATCH Commun. Math. Comput. Chem. **50** (2004) 83–92.
- [18] I. Gutman, P. Hansen and H. Mélot, *Variable neighborhood search for extremal graphs. 10. Comparison of irregularity indices for chemical trees*, J. Chem. Inf. Model. **45** (2005) 222–230.
doi:10.1021/ci0342775
- [19] R. Hammack, W. Imrich and S. Klavžar, *Handbook of Product Graphs* (CRC Press, Boca Raton, FL, 2011).
- [20] P. Hansen and H. Mélot, *Variable neighborhood search for extremal graphs. 9. Bounding the irregularity of a graph*, DIMACS Ser. Discrete Math. Theoret. Comput. Sci. **69** (2005) 253–264.
- [21] M.A. Henning and D. Rautenbach, *On the irregularity of bipartite graphs*, Discrete Math. **307** (2007) 1467–1472.

- [22] D.E. Jackson and R. Entringer, *Totally segregated graphs*, Congr. Numer. **55** (1986) 159–165.
- [23] D.J. Miller, *The categorical product of graphs*, Canad. J. Math. **20** (1968) 1511–1521.
doi:10.4153/CJM-1968-151-x
- [24] S. Nikolić, G. Kovačević, A. Miličević and N. Trinajstić, *The Zagreb indices 30 years after*, Croat. Chem. Acta **76** (2003) 113–124.
- [25] N. Trinajstić, S. Nikolić, A. Miličević and I. Gutman, *On Zagreb indices*, Kem. Ind. **59** (2010) 577–589.
- [26] P.M. Weichsel, *The Kronecker product of graphs*, Proc. Amer. Math. Soc. **13** (1962) 47–52.
doi:10.4153/CJM-1968-151-x
- [27] V. Yegnanarayanan, P.R. Thiripurasundari and T. Padmavathy, *On some graph operations and related applications*, Electron. Notes Discrete Math. **33** (2009) 123–130.
doi:10.1016/j.endm.2009.03.018
- [28] B. Zhou, *Remarks on Zagreb indices*, MATCH Commun. Math. Comput. Chem. **57** (2007) 591–596.
- [29] B. Zhou and I. Gutman, *Further properties of Zagreb indices*, MATCH Commun. Math. Comput. Chem **54** (2005) 233–239.

Received 3 September 2012

Revised 22 January 2013

Accepted 11 March 2013