Discussiones Mathematicae
Graph Theory 34 (2014) 233–247
doi:10.7151/dmgt.1736

THE DEPRESSION OF A GRAPH AND k-KERNELS

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Abstract

An *edge ordering* of a graph G is an injection $f : E(G) \to \mathbb{R}$, the set of real numbers. A path in G for which the edge ordering f increases along its edge sequence is called an f-*ascent*; an f-ascent is *maximal* if it is not contained in a longer f-ascent. The *depression* of G is the smallest integer k such that any edge ordering f has a maximal f-ascent of length at most k. A k-kernel of a graph G is a set of vertices $U \subseteq V(G)$ such that for any edge ordering f of G there exists a maximal f-ascent of length at most k which neither starts nor ends in U. Identifying a k-kernel of a graph G enables one to construct an infinite family of graphs from G which have depression at most k. We discuss various results related to the concept of k-kernels, including an improved upper bound for the depression of trees.

Keywords: edge ordering of a graph, increasing path, monotone path, depression.

2010 Mathematics Subject Classification: 05C78, 05C38.

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Received 5 November 2012
Revised 26 February 2013
Accepted 26 February 2013