

## MOTION PLANNING IN CARTESIAN PRODUCT GRAPHS

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### Abstract

Let  $G$  be an undirected graph with  $n$  vertices. Assume that a robot is placed on a vertex and  $n - 2$  obstacles are placed on the other vertices. A vertex on which neither a robot nor an obstacle is placed is said to have a hole. Consider a single player game in which a robot or obstacle can be moved to adjacent vertex if it has a hole. The objective is to take the robot to a fixed destination vertex using minimum number of moves. In general, it is not necessary that the robot will take a shortest path between the source and destination vertices in graph  $G$ . In this article we show that the path traced by the robot coincides with a shortest path in case of Cartesian product graphs. We give the minimum number of moves required for the motion planning problem in Cartesian product of two graphs having girth 6 or more. A result that we prove in the context of Cartesian product of  $P_n$  with itself has been used earlier to develop an approximation algorithm for  $(n^2 - 1)$ -puzzle.

**Keywords:** robot motion in a graph, Cartesian product of graphs.

**2010 Mathematics Subject Classification:** 05C85, 05C75, 68R10, 91A43.

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