

A NOTE ON VERTEX COLORINGS OF PLANE GRAPHS¹

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Abstract

Given an integer valued weighting of all elements of a 2-connected plane graph G with vertex set V , let $c(v)$ denote the sum of the weight of $v \in V$ and of the weights of all edges and all faces incident with v . This vertex coloring of G is *proper* provided that $c(u) \neq c(v)$ for any two adjacent vertices u and v of G . We show that for every 2-connected plane graph there is such a proper vertex coloring with weights in $\{1, 2, 3\}$. In a special case, the value 3 is improved to 2.

Keywords: plane graph, vertex coloring.

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REFERENCES

- [1] L. Addario-Berry, K. Dalal, C. McDiarmid, B.A. Reed and A. Thomason, *Vertex-coloring edge-weightings*, *Combinatorica* **27** (2007) 1–12.
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- [2] L. Addario-Berry, K. Dalaland and B.A. Reed, *Degree constrained subgraphs*, Discrete Appl. Math. **156** (2008) 1168–1174.
doi:10.1016/j.dam.2007.05.059
- [3] K. Appel and W. Haken, *Every planar map is four-colorable, I. Discharging*, Illinois J. Math. **21** (1977) 429–490.
- [4] M. Axenovich, J. Harant, J. Przybyło, R. Soták and M. Voigt, *A note on adjacent vertex distinguishing colorings number of graphs*, Electron. J. Combin. (submitted).
- [5] M. Bača, S. Jendrol', M. Miller and J. Ryan, *On irregular total labellings*, Discrete Math. **307** (2007) 1378–1388.
doi:10.1016/j.disc.2005.11.075
- [6] T. Bartnicki, B. Bosek, S. Czerwiński, J. Grytczuk, G. Matecki and W. Żelazny, *Additive colorings of planar graphs*, Graphs Combin. **30** (2014) 1087–1098.
doi:10.1007/s00373-013-1331-y
- [7] J.A. Bondy and U.S.R. Murty, Graph Theory (Springer, 2008).
- [8] G. Chartrand, M.S. Jacobson, L. Lehel, O.R. Oellermann, S. Ruiz and F. Saba, *Irregular networks*, Congr. Numer. **64** (1988) 187–192.
- [9] S. Czerwiński, J. Grytczuk and W. Żelazny, *Lucky labelings of graphs*, Inform. Process. Lett. **109** (2009) 1078–1081.
doi:10.1016/j.ipl.2009.05.011
- [10] R. Diestel, Graph Theory (Springer, 2000).
- [11] A. Frieze, R.J. Gould, M. Karoński and F. Pfender, *On graph irregularity strenght*, J. Graph Theory **41** (2002) 120–137.
doi:10.1002/jgt.10056
- [12] S. Jendrol' and P. Šugerek, *A note on face coloring entire weightings of plane graphs*, Discuss. Math. Graph Theory **34** (2014) 421–426.
doi:10.7151/dmgt.1738
- [13] T.R. Jensen and B. Toft, Graph Coloring Problems (Wiley, 1995).
- [14] M. Kalkowski, *A note on 1, 2-conjecture*, Electron. J. Combin. (to appear).
- [15] M. Kalkowski, M. Karoński and F. Pfender, *Vertex-coloring edge-weightings: towards the 1-2-3-conjecture*, J. Combin. Theory (B) **100** (2010) 347–349.
doi:10.1016/j.jctb.2009.06.002
- [16] M. Karoński and T. Łuczak, A. Thomason, *Edge weights and vertex colors*, J. Combin. Theory (B) **91** (2004) 151–157.
doi:10.1016/j.jctb.2003.12.001
- [17] J. Przybyło and M. Woźniak, *On 1, 2 conjecture*, Discrete Math. Theor. Comput. Sci. **12** (2010) 101–108.
- [18] T. Wang and Q. Yu, *On vertex-coloring 13-edge-weighting*, Front. Math. China **3** (2008) 1–7.
doi:10.1007/s11461-008-0009-8

- [19] W. Wang and X. Zhu, *Entire coloring of plane graphs*, J. Combin. Theory (B) **101** (2011) 490–501.
doi:10.1016/j.jctb.2011.02.006

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