

ON THE COMPLEXITY OF THE 3-KERNEL PROBLEM IN SOME CLASSES OF DIGRAPHS¹

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Abstract

Let D be a digraph with the vertex set $V(D)$ and the arc set $A(D)$. A subset N of $V(D)$ is k -independent if for every pair of vertices $u, v \in N$, we have $d(u, v), d(v, u) \geq k$; it is l -absorbent if for every $u \in V(D) - N$ there exists $v \in N$ such that $d(u, v) \leq l$. A k -kernel of D is a k -independent and $(k - 1)$ -absorbent subset of $V(D)$. A 2-kernel is called a *kernel*.

It is known that the problem of determining whether a digraph has a kernel (“the kernel problem”) is NP-complete, even in quite restricted families of digraphs. In this paper we analyze the computational complexity of the corresponding 3-kernel problem, restricted to three natural families of digraphs.

As a consequence of one of our main results we prove that the kernel problem remains NP-complete when restricted to 3-colorable digraphs.

Keywords: kernel, 3-kernel, NP-completeness, multipartite tournament, cyclically 3-partite digraphs, k -quasi-transitive digraph.

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REFERENCES

- [1] J. Bang-Jensen and G. Gutin, *Digraphs* (Springer-Verlag, Berlin Heidelberg New York, 2002).
- [2] J. Bang-Jensen and J. Huang, *Quasi-transitive digraphs*, *J. Graph Theory* **20** (1995) 141–161.
doi:10.1002/jgt.3190200205

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- [3] C. Berge, *Graphs* (North-Holland, Amsterdam, 1985).
- [4] C. Berge and P. Duchet, *Recent problems and results about kernels in directed graphs*, *Discrete Math.* **86** (1990) 27–31.
doi:10.1016/0012-365X(90)90346-J
- [5] J.A. Bondy and U.S.R. Murty, *Graph Theory* (Springer-Verlag, Berlin Heidelberg New York, 2008).
- [6] V. Chvátal, *On the computational complexity of finding a kernel*, Technical Report Centre de Recherches Mathématiques, Université de Montréal CRM-300 (1973).
- [7] A.S. Fraenkel, *Planar kernel and Grundy with $d \leq 3, d^+ \leq 2, d^- \leq 2$ are NP-complete*, *Discrete Appl. Math.* **3** (1981) 257–262.
doi:10.1016/0166-218X(81)90003-2
- [8] H. Galeana-Sánchez, I.A. Goldfeder and I. Urrutia, *On the structure of 3-quasi-transitive digraphs*, *Discrete Math.* **310** (2010) 2495–2498.
doi:10.1016/j.disc.2010.06.008
- [9] H. Galeana-Sánchez and C. Hernández-Cruz, *k-kernels in multipartite tournaments*, *AKCE Int. J. Graphs Comb.* **8** (2011) 181–198.
- [10] H. Galeana-Sánchez and C. Hernández-Cruz, *On the existence of k-kernels digraphs and in weighted digraphs*, *AKCE Int. J. Graphs Comb.* **7** (2010) 201–215.
- [11] H. Galeana-Sánchez and C. Hernández-Cruz, *Cyclically k-partite digraphs and k-kernels*, *Discuss. Math. Graph Theory* **31** (2011) 63–78.
doi:10.7151/dmgt.1530
- [12] H. Galeana-Sánchez and C. Hernández-Cruz, *k-kernels in generalizations of transitive digraphs*, *Discuss. Math. Graph Theory* **31** (2011) 293–312.
doi:10.7151/dmgt.1546
- [13] H. Galeana-Sánchez, C. Hernández-Cruz and M.A. Juárez-Camacho, *On the existence and number of (k+1)-kings in k-quasi-transitive digraphs*, *Discrete Math.* **313** (2013) 2582–2591.
- [14] C. Hernández-Cruz and H. Galeana-Sánchez, *k-kernels in k-transitive and k-quasi-transitive digraphs*, *Discrete Math.* **312** (2012) 2522–2530.
doi:10.1016/j.disc.2012.05.005
- [15] M. Kwaśnik, A. Włoch and I. Włoch, *Some remarks about (k, l)-kernels in directed and undirected graphs*, *Discuss. Math.* **13** (1993) 29–37.
- [16] M. Richardson, *On weakly ordered systems*, *Bull. Amer. Math. Soc.* **52(2)** (1946) 113–116.
doi:10.1090/S0002-9904-1946-08518-3
- [17] A. Sánchez-Flores, *A counterexample to a generalization of Richardson's theorem*, *Discrete Math.* **65** (1987) 319–320.
doi:10.1016/0012-365X(87)90064-1

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