

NOTE

STAR-CYCLE FACTORS OF GRAPHS

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Abstract

A spanning subgraph F of a graph G is called a *star-cycle factor* of G if each component of F is a star or cycle. Let G be a graph and $f : V(G) \rightarrow \{1, 2, 3, \dots\}$ be a function. Let $W = \{v \in V(G) : f(v) = 1\}$. Under this notation, it was proved by Berge and Las Vergnas that G has a star-cycle factor F with the property that (i) if a component D of F is a star with center v , then $\deg_F(v) \leq f(v)$, and (ii) if a component D of F is a cycle, then $V(D) \subseteq W$ if and only if $iso(G - S) \leq \sum_{x \in S} f(x)$ for all $S \subset V(G)$, where $iso(G - S)$ denotes the number of isolated vertices of $G - S$. They proved this result by using circulation theory of flows and fractional factors of graphs. In this paper, we give an elementary and short proof of this theorem.

Keywords: star factor, cycle factor, star-cycle factor, factor of graph.

2010 Mathematics Subject Classification: 05C70.

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¹The author was supported by Japan Society for the Promotion of Science, Grant-in-Aid for Scientific Research (C).

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Received 28 August 2012
Revised 13 December 2012
Accepted 28 December 2012