

THE MINIMUM SPECTRAL RADIUS OF
SIGNLESS LAPLACIAN OF GRAPHS
WITH A GIVEN CLIQUE NUMBER

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Abstract

In this paper we observe that the minimal signless Laplacian spectral radius is obtained uniquely at the kite graph $PK_{n-\omega,\omega}$ among all connected graphs with n vertices and clique number ω . In addition, we show that the spectral radius μ of $PK_{m,\omega}$ ($m \geq 1$) satisfies

$$\frac{1}{2}(2\omega - 1 + \sqrt{4\omega^2 - 12\omega + 17}) \leq \mu \leq 2\omega - 1.$$

More precisely, for $m > 1$, μ satisfies the equation

$$\mu - \omega - \frac{\omega - 1}{\mu - 2\omega + 3} = a_m \sqrt{\mu^2 - 4\mu} + \frac{1}{t_1},$$

where $a_m = \frac{1}{1-t_1^{2m+3}}$ and $t_1 = \frac{\mu-2+\sqrt{(\mu-2)^2-4}}{2}$. At last the spectral radius $\mu(PK_{\infty,\omega})$ of the infinite graph $PK_{\infty,\omega}$ is also discussed.

Keywords: clique number, kite graph, signless Laplacian, spectral radius.

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