

PACKING THE HYPERCUBE

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Abstract

Let G be a graph that is a subgraph of some n -dimensional hypercube Q_n . For sufficiently large n , Stout [20] proved that it is possible to pack vertex-disjoint copies of G in Q_n so that any proportion $r < 1$ of the vertices of Q_n are covered by the packing. We prove an analogous theorem for edge-disjoint packings: For sufficiently large n , it is possible to pack edge-disjoint copies of G in Q_n so that any proportion $r < 1$ of the edges of Q_n are covered by the packing.

Keywords: hypercube, packing, decomposition.

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