

## ON THE NUMBERS OF CUT-VERTICES AND END-BLOCKS IN 4-REGULAR GRAPHS

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### Abstract

A *cut-vertex* in a graph  $G$  is a vertex whose removal increases the number of connected components of  $G$ . An *end-block* of  $G$  is a block with a single cut-vertex. In this paper we establish upper bounds on the numbers of end-blocks and cut-vertices in a 4-regular graph  $G$  and claw-free 4-regular graphs. We characterize the extremal graphs achieving these bounds.

**Keywords:** 4-regular graph, claw-free, cut-vertices, end-blocks.

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