

STRONG CHROMATIC INDEX OF PLANAR GRAPHS WITH LARGE GIRTH¹

GERARD JENNHWA CHANG¹²³, MICKAEL MONTASSIER⁴,

ARNAUD PÊCHER⁵ AND ANDRÉ RASPAUD⁵

¹*Department of Mathematics*
and

²*Taida Institute for Mathematical Sciences*
National Taiwan University, Taipei 10617, Taiwan

³*National Center for Theoretical Sciences, Taipei Office, Taiwan*

⁴*Universit Montpellier 2, CNRS-LIRMM, UMR5506*
161 rue Ada, 34095 Montpellier Cedex 5, France

⁵*LaBRI - University of Bordeaux*
351 cours de la Liberation, 33405 Talence Cedex, France

e-mail: raspaud@labri.fr

Abstract

Let $\Delta \geq 4$ be an integer. In this note, we prove that every planar graph with maximum degree Δ and girth at least $10\Delta + 46$ is strong $(2\Delta - 1)$ -edge-colorable, that is best possible (in terms of number of colors) as soon as G contains two adjacent vertices of degree Δ . This improves [6] when $\Delta \geq 6$.

Keywords: planar graphs, edge coloring, 2-distance coloring, strong edge-coloring.

2010 Mathematics Subject Classification: 05C15.

REFERENCES

- [1] L.D. Andersen, *The strong chromatic index of a cubic graph is at most 10*, *Discrete Math.* **108** (1992) 231–252.
doi:10.1016/0012-365X(92)90678-9
- [2] K. Appel and W. Haken, *Every planar map is four colorable. Part I. Discharging*, *Illinois J. Math.* **21** (1977) 429–490.
- [3] K. Appel and W. Haken, *Every planar map is four colorable. Part II. Reducibility*, *Illinois J. Math.* **21** (1977) 491–567.

- [4] C.L. Barrett, G. Istrate, V.S.A. Kumar, M.V. Marathe, S. Thite, and S. Thulasidasan, *Strong edge coloring for channel assignment in wireless radio networks*, in: Proc. of the 4th Annual IEEE International Conference on Pervasive Computing and Communications Workshops (2006) 106–110.
- [5] N. Biggs, *Some odd graph theory*, Annals New York Academy of Sciences **319** (1979) 71–81.
- [6] O.V. Borodin and A.O. Ivanova, *Precise upper bound for the strong edge chromatic number of sparse planar graphs*, Discuss. Math. Graph Theory **33** (2013) 759–770. doi:10.7151/dmgt.1708
- [7] D.W. Cranston, *Strong edge-coloring of graphs with maximum degree 4 using 22 colors*, Discrete Math. **306** (2006) 2772–2778. doi:10.1016/j.disc.2006.03.053
- [8] P. Erdős, *Problems and results in combinatorial analysis and graph theory*, Discrete Math. **72** (1988) 81–92. doi:10.1016/0012-365X(88)90196-3
- [9] P. Erdős and J. Nešetřil, *Problem*, in: Irregularities of Partitions, G. Halász and V.T. Sós (Eds.) (Springer, Berlin, 1989) 162–163.
- [10] R.J. Faudree, A. Gyárfas, R.H. Schelp and Zs. Tuza, *The strong chromatic index of graphs*, Ars Combin. **29B** (1990) 205–211.
- [11] J.L. Fouquet and J.L. Jolivet, *Strong edge-coloring of graphs and applications to multi-k-gons*, Ars Combin. **16** (1983) 141–150.
- [12] J.L. Fouquet and J.L. Jolivet, *Strong edge-coloring of cubic planar graphs*, Progress in Graph Theory (Waterloo 1982), (1984) 247–264.
- [13] H. Grötzsch, *Ein Dreifarbensatz für Dreiecksfreie Netze auf der Kugel*, Math.-Nat. Reihe **8** (1959) 109–120.
- [14] H. Hocquard, P. Ochem and P. Valicov, *Strong edge coloring and induced matchings*, LaBRI Research Report, 2011. http://hal.archives-ouvertes.fr/hal-00609454_v1/
- [15] P. Horák, H. Qing, and W.T. Trotter, *Induced matchings in cubic graphs*, J. Graph Theory **17** (1993) 151–160. doi:10.1002/jgt.3190170204
- [16] M. Mahdian, *The strong chromatic index of graphs*, Master Thesis (University of Toronto, Canada, 2000).
- [17] M. Molloy and B. Reed, *A bound on the strong chromatic index of a graph*, J. Combin. Theory (B) **69** (1997) 103–109. doi:10.1006/jctb.1997.1724
- [18] T. Nandagopal, T. Kim, X. Gao and V. Bharghavan, *Achieving MAC layer fairness in wireless packet networks*, in: Proc. 6th ACM Conf. on Mobile Computing and Networking (2000) 87–98.

- [19] J. Nešetřil, A. Raspaud and A. Sopena, *Colorings and girth of oriented planar graphs*, Discrete Math. **165–166** (1997) 519–530.
doi:10.1016/S0012-365X(96)00198-7
- [20] S. Ramanathan, *A unified framework and algorithm for (T/F/C) DMA channel assignment in wireless networks*, in: Proc. IEEE INFOCOM'97 (1997) 900–907.
doi:10.1109/INFCOM.1997.644573
- [21] S. Ramanathan and E.L. Lloyd, *Scheduling algorithms for multi-hop radio networks*, in: IEEE/ACM Trans. Networking **2** (1993) 166–177.
doi:10.1109/90.222924
- [22] D.P. Sanders and Y. Zhao, *Planar graphs of maximum degree seven are Class 1*, J. Combin. Theory (B) **83** (2001) 201–212.
doi:1006/jctb.2001.2047
- [23] V.G. Vizing, *On an estimate of the chromatic class of a p -graph*, Diskret. Analiz. **3** (1964) 25–30.

Received 5 April 2013
Revised 30 October 2013
Accepted 30 October 2013