

SOME REMARKS ON THE STRUCTURE OF STRONG k -TRANSITIVE DIGRAPHS¹

CÉSAR HERNÁNDEZ-CRUZ

AND

JUAN JOSÉ MONTELLANO-BALLESTEROS

Instituto de Matemáticas
Universidad Nacional Autónoma de México
Ciudad Universitaria, México, D.F., C.P. 04510, México

e-mail: cesar@matem.unam.mx
juancho@matem.unam.mx

Abstract

A digraph D is k -transitive if the existence of a directed path (v_0, v_1, \dots, v_k) , of length k implies that $(v_0, v_k) \in A(D)$. Clearly, a 2-transitive digraph is a transitive digraph in the usual sense. Transitive digraphs have been characterized as compositions of complete digraphs on an acyclic transitive digraph. Also, strong 3 and 4-transitive digraphs have been characterized.

In this work we analyze the structure of strong k -transitive digraphs having a cycle of length at least k . We show that in most cases, such digraphs are complete digraphs or cycle extensions. Also, the obtained results are used to prove some particular cases of the Laborde-Payan-Xuong Conjecture.

Keywords: digraph, transitive digraph, k -transitive digraph, quasi-transitive digraph, k -quasi-transitive digraph, Laborde-Payan-Xuong Conjecture.

2010 Mathematics Subject Classification: 05C20, 05C75.

REFERENCES

- [1] J. Bang-Jensen and G. Gutin, *Digraphs. Theory, Algorithms and Applications* (Springer-Verlag Berlin, Heidelberg New York, 2002).
- [2] R. Diestel, *Graph Theory 3rd Edition* (Springer-Verlag Berlin, Heidelberg New York, 2005).

¹Research supported by PAPIIT-México project IN101912.

- [3] H. Galeana-Sánchez and C. Hernández-Cruz, *k-kernels in k-transitive and k-quasi-transitive digraphs*, Discrete Math. **312** (2012) 2522–2530.
doi:10.1016/j.disc.2012.05.005
- [4] A. Ghouila-Houri, *Caractérisation des graphes non orientés dont on peut orienter les arrêtes de manière à obtenir le graphe d'une relation d'ordre*, C. R. Acad. Sci. Paris **254** (1962) 1370–1371.
- [5] C. Hernández-Cruz, *3-transitive digraphs*, Discuss. Math. Graph Theory **32** (2012) 205–219.
doi:10.7151/dmgt.1613
- [6] C. Hernández-Cruz, *4-transitive digraphs I: The structure of strong 4-transitive digraphs*, Discuss. Math. Graph Theory **33** (2013) 247–260.
doi:10.7151/dmgt.1645
- [7] J.M. Laborde, C. Payan and N.H. Xuong, *Independent sets and longest directed paths in digraphs*, in: Graphs and other Combinatorial Topics, Prague, M. Fiedler (Ed(s)), (Teubner, Leipzig, 1983) 173–177.
- [8] R. Wang, *A conjecture on k-transitive digraphs*, Discrete Math. **312** (2012) 1458–1460.
doi:10.1016/j.disc.2012.01.011
- [9] R. Wang and S. Wang, *Underlying graphs of 3-quasi-transitive digraphs and 3-transitive digraphs*, Discuss. Math. Graph Theory **33** (2013) 429–436.
doi:10.7151/dmgt.1680

Received 7 May 2012
Revised 15 July 2013
Accepted 6 August 2013