

DOWNHILL DOMINATION IN GRAPHS

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Abstract

A path $\pi = (v_1, v_2, \dots, v_{k+1})$ in a graph $G = (V, E)$ is a *downhill path* if for every i , $1 \leq i \leq k$, $\deg(v_i) \geq \deg(v_{i+1})$, where $\deg(v_i)$ denotes the degree of vertex $v_i \in V$. The *downhill domination number* equals the minimum cardinality of a set $S \subseteq V$ having the property that every vertex $v \in V$ lies on a downhill path originating from some vertex in S . We investigate downhill domination numbers of graphs and give upper bounds. In particular, we show that the downhill domination number of a graph is at most half its order, and that the downhill domination number of a tree is at most one third its order. We characterize the graphs obtaining each of these bounds.

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REFERENCES

- [1] T.W. Haynes, S.T. Hedetniemi, J. Jamieson and W. Jamieson, *Downhill and uphill domination in graphs*, submitted for publication (2013).
- [2] P. Hall, *On representation of subsets*, J. London Math. Soc. **10** (1935) 26–30.
- [3] T.W. Haynes, S.T. Hedetniemi and P.J. Slater, *Fundamentals of Domination in Graphs* (Marcel Dekker, 1998).
- [4] J.D. Hedetniemi, S.M. Hedetniemi, S.T. Hedetniemi and T. Lewis, *Analyzing graphs by degrees*, AKCE Int. J. Graphs Comb., to appear.
- [5] O. Ore, *Theory of Graphs* (Amer. Math. Soc. Colloq. Publ. 38, 1962).

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