

ON THE EXISTENCE OF (k, l) -KERNELS IN INFINITE DIGRAPHS: A SURVEY

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Abstract

Let D be a digraph, $V(D)$ and $A(D)$ will denote the sets of vertices and arcs of D , respectively.

A (k, l) -kernel N of D is a k -independent (if $u, v \in N$, $u \neq v$, then $d(u, v), d(v, u) \geq k$) and l -absorbent (if $u \in V(D) - N$ then there exists $v \in N$ such that $d(u, v) \leq l$) set of vertices. A k -kernel is a $(k, k - 1)$ -kernel.

This work is a survey of results proving sufficient conditions for the existence of (k, l) -kernels in infinite digraphs. Despite all the previous work in this direction was done for $(2, 1)$ -kernels, we present many original results concerning (k, l) -kernels for distinct values of k and l .

The original results are sufficient conditions for the existence of (k, l) -kernels in diverse families of infinite digraphs. Among the families that we study are: transitive digraphs, quasi-transitive digraphs, right/left pretransitive digraphs, cyclically k -partite digraphs, κ -strong digraphs, k -transitive digraphs, k -quasi-transitive digraphs.

Keywords: kernel, k -kernel, (k, l) -kernel, infinite digraph.

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