

ON THE INDEPENDENCE NUMBER OF EDGE CHROMATIC CRITICAL GRAPHS*

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Abstract

In 1968, Vizing conjectured that for any edge chromatic critical graph $G = (V, E)$ with maximum degree Δ and independence number $\alpha(G)$, $\alpha(G) \leq \frac{|V|}{2}$. It is known that $\alpha(G) < \frac{3\Delta-2}{5\Delta-2}|V|$. In this paper we improve this bound when $\Delta \geq 4$. Our precise result depends on the number n_2 of 2-vertices in G , but in particular we prove that $\alpha(G) \leq \frac{3\Delta-3}{5\Delta-3}|V|$ when $\Delta \geq 5$ and $n_2 \leq 2(\Delta - 1)$.

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