

## ON UNIQUE MINIMUM DOMINATING SETS IN SOME CARTESIAN PRODUCT GRAPHS

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### Abstract

Unique minimum vertex dominating sets in the Cartesian product of a graph with a complete graph are considered. We first give properties of such sets when they exist. We then show that when the first factor of the product is a tree, consideration of the tree alone is sufficient to determine if the product has a unique minimum dominating set.

**Keywords:** vertex domination, graph products, trees.

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