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SURVEY OF CERTAIN VALUATIONS OF GRAPHS

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Abstract

The study of valuations of graphs is a relatively young part of graph theory. In this article we survey what is known about certain graph valuations, that is, labeling methods: antimagic labelings, edge-magic total labelings and vertex-magic total labelings.

Keywords: (a, d)-antimagic labeling, (a, d)-face antimagic labeling, edge-magic total labeling, vertex-magic total labeling.

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1 Introduction

Several decades ago Ringel, Kotzig and Rosa (see [27, 29]) discovered a simple, very interesting and purely combinatorial classification of all finite trees which can be labeled in a way they called *graceful*. That they could not prove their discovery was a great pity for them but an immense stroke of luck for graph theory because a new topic was born; in the period immediately after the publication of the conjecture, most graph theorists dealing with graphs and integers wanted to prove or disprove the Ringel-Kotzig conjecture (see [7]). Since the conjecture resisted all the attempts at a solution, many new interesting labelings of graphs were introduced and dealt with, labelings which are quite different from the graceful labeling. Unfortunately, it turned out that it was not any easier to determine or characterize the set of all graphs labeled in the new ways than it was to settle the Ringel-Kotzig conjecture. In the intervening 32 years, well over 250 papers on this topic have appeared. Summarizing and describing the contemporary situation in the theory of labeling graphs, we can state that for about 32 years it has been an interesting and important central problem to find new classes of finite (or infinite) graphs labeled in certain ways or to construct new types of labelings, knowing that it would be impossible to develop a closed theory giving a complete characterization of the set of graphs permitting a new labeling (see [10, 16]).

Labeled graphs serve as useful models for a broad range of applications such as: coding theory, x-ray crystallography, radar, astronomy, circuit design and communication networks (see [8]).

In this article, we survey what is known about certain graph labeling methods. In particular we discuss antimagic labelings, edge-magic total labelings and vertex-magic total labelings.

Many other researchers have investigated different forms of magic graphs. For examples, see Borowiecki and Quintas [13], Jeurissen [18], Jezný and Trenkler [19], Sedláček [30], Stewart [31].

All graphs considered in this paper are finite, simple, and undirected. The graph G has vertex-set $V(G)$ and edge-set $E(G)$ and we let $|V| = v$ and $|E| = e$. A general reference for graph-theoretic notions is [34].

2 (a, d)-Antimagic Labelings

The concept of an antimagic labeling was introduced by Hartsfield and Ringel [17]. An *antimagic graph* is a graph whose edges can be labeled with the integers $1, 2, ..., e$ so that the sum of the labels at any given vertex is different from the sum of the labels at any other vertex, that is, no two vertices have the same sum. Hartsfield and Ringel conjecture that every tree other than K_2 is antimagic and, more strongly, that every connected graph other than K_2 is antimagic.

The *weight* $w(x)$ of a vertex $x \in V(G)$ under an edge labeling $f : E \rightarrow \{1, 2, ..., e\}$ is the sum of values $f(xy)$ assigned to all edges incident to a given vertex x. Bodendiek and Walther [9] defined the concept of an (a, d)-antimagic graph as a special case of an antimagic graph as follows.

A connected graph $G = (V, E)$ is said to be (a, d)-*antimagic* if there exist positive integers a, d and a bijection $f : E \rightarrow \{1, 2, ..., e\}$ such that the induced mapping $g_f : V(G) \rightarrow W$ is also a bijection, where $W = \{w(x) : x \in V(G)\} = \{a, a + d, a + 2d, ..., a + (v - 1)d\}$ is the set of the weights of vertices.

Bodendiek and Walther [10] showed that the theory of Diophantine equations and other concepts of number theory can be applied to determine the set of all connected (a, d)-antimagic graphs. For special graphs called *parachutes*, (a, d)-antimagic labelings are described in [11, 12].

The *prism* D_n, $n \geq 3$, is a trivalent graph which can be defined as the Cartesian product $P_2 \times C_n$ of a path on two vertices with a cycle on n vertices. The prism can also be defined as the Cayley graph of the dihedral group of order $2n$ (using a rotation and a reflection as generators, see [15]).

Bodendiek and Walther [10] conjecture that D_n, $n \equiv 0 \pmod 2$, is $(\frac{7n+4}{2}, 1)$-antimagic and D_n, $n \equiv 1 \pmod 2$, is $(\frac{5n+5}{2}, 2)$-antimagic. The necessary conditions for (a, d)-antimagic labeling of the prism D_n are given in [1], namely, if D_n is (a, d)-antimagic then

- for n even: either $d = 1$ and $a = \frac{7n+4}{2}$ or $d = 3$ and $a = \frac{3n+6}{2}$; and
- for n odd: either $d = 2$ and $a = \frac{5n+5}{2}$ or $d = 4$ and $a = \frac{n+7}{2}$.

Proofs of the conjectures of Bodendiek and Walther are given in [1] where it is also shown that D_n, $n \equiv 0 \pmod 2$, is $(\frac{3n+6}{2}, 3)$-antimagic and it is conjectured that if n is odd, $n \geq 7$, then the prism D_n is $(\frac{n+7}{2}, 4)$-antimagic.

The *antiprism* A_n, $n \geq 3$, is the plane regular graph of degree 4 (an Archimedean convex polytope). In particular, A_3 is the octahedron. Bača [2] showed that (a, d)-antimagic labelings of antiprisms do not exist for all values of (a, d) other than $(6n + 3, 2)$, $(4n + 4, 4)$, and $(2n + 5, 6)$. In the same paper, he also gave the following labelings of A_n:

- $(6n + 3, 2)$-antimagic labeling for $n \geq 3$, $n \not\equiv 2 \pmod 4$; and
- $(4n + 4, 4)$-antimagic labeling for $n \geq 3$, $n \not\equiv 2 \pmod 4$.

In [25] are given a $(6n+3, 2)$-antimagic labeling and a $(4n+4, 4)$-antimagic labeling of A_n for every even n, thereby proving two of Bača's three conjectures listed in [2]. To characterize (a, d)-antimagic antiprisms, it remains to prove the following conjecture:

- for $n \geq 4$ the antiprism A_n has an $(2n+5, 6)$-antimagic labeling.

A *generalized Petersen graph* $P(n, m)$, $1 \leq m < \frac{n}{2}$, consists of an outer n-cycle $y_1, y_2, ..., y_n$, a set of n spokes $y_i x_i$, $1 \leq i \leq n$, and n inner edges $x_i x_{i+m}$, $1 \leq i \leq n$, with indices taken modulo n. The standard Petersen graph is the instance $P(5, 2)$.

Generalized Petersen graphs were first defined by Watkins [33]. We note that the prism D_n is the generalized Petersen graph $P(n, 1)$. The necessary conditions for (a, d)-antimagicness of $P(n, m)$ are the same as for the prism D_n.

In [5] it is proved that the generalized Petersen graph $P(n, m)$ is $(a, 1)$-antimagic if and only if n is even, $n \geq 4$, $m \leq \frac{n}{2} - 1$ and $a = \frac{7n+4}{2}$. Miller and Bača [24] showed that if $n \equiv 0 \pmod 4$ then $P(n, 2)$ is $(\frac{3n+6}{2}, 3)$-antimagic.

We conjecture that the generalized Petersen graph $P(n, m)$ is (a, d)-antimagic for all feasible values of a and d. More specifically, we put forward the following three conjectures:

- If n is even, $n \geq 6$ and $2 \leq m \leq \frac{n}{2} - 1$, then $P(n, m)$ is $(\frac{3n}{2} + 3, 3)$-antimagic.
- If n is odd, $n \geq 5$ and $2 \leq m \leq \frac{n-1}{2}$, then $P(n, m)$ is $(\frac{5n+5}{2}, 2)$-antimagic.
- If n is odd, $n \geq 7$ and $1 \leq m \leq \frac{n-1}{2}$, then $P(n, m)$ is $(\frac{n+7}{2}, 4)$-antimagic.

3 (a, d)-Face Antimagic Labelings

A graph is said to be *plane* if it is drawn on the Euclidean plane in such a way that edges do not cross each other except at vertices of the graph. Assume that all plane graphs considered in this paper possess no vertices of degree one. For a plane graph $G = (V, E, F)$, it makes sense to consider its faces, including the unique face of infinite area. Let $F(G)$ be the face set and $|F(G)|$ be the number of the faces of G.

Now let us define the weight of a face and (a, d)-face antimagic labeling of the plane graph $G = (V, E, F)$. The *weight* $w^*(f)$ of a face $f \in F(G)$ under an edge labeling $g : E(G) \to \{1, 2, ..., |E(G)|\}$ is the sum of the labels of edges surrounding that face.

A connected plane graph $G = (V, E, F)$ is said to be (a, d)-*face antimagic* if there exist positive integers a, d and a bijection $g : E(G) \to \{1, 2, ..., |E(G)|\}$ such that the induced mapping $w^*_g : F(G) \to W$ is also a bijection, where $W = \{w^*(f) : f \in F(G)\} = \{a, a + d, ..., a + (|F(G)| - 1)d\}$ is the set of weights of faces. If $G = (V, E, F)$ is (a, d)-face antimagic and $g : E(G) \to \{1, 2, ..., |E(G)|\}$ is a corresponding bijective mapping of G then g is said to be an (a, d)-face antimagic labeling of G.

Clearly, an (a, d)-face antimagic labeling of the graph G of a convex polytope is equivalent to (a, d)-antimagic labeling a dual graph G^*. The plane dual graph D^*_n of prism D_n is the graph of a bipyramid. This implies the following lemmas.

Lemma 1. *If n is even, $n \geq 4$, then the bipyramid D^*_n has a $(\frac{7n+4}{2}, 1)$-face antimagic labeling and $(\frac{3n+6}{2}, 3)$-face antimagic labeling.*

Lemma 2. *If n is odd, $n \geq 3$, then the bipyramid D^*_n has a $(\frac{5n+5}{2}, 2)$-face antimagic labeling.*

Since the plane dual graph A^*_n of the antiprism A_n is the graph of a quasi-bipyramid, then we have

Lemma 3. *For $n \geq 3$, the quasibipyramid A^*_n is $(6n + 3, 2)$-face antimagic and $(4n + 4, 4)$-face antimagic.*

Let $I = \{1, 2, ..., n\}$ be an index set. Let us denote the vertex set of the prism D_n by $V(D_n) = \{x_{j,i} : i \in I \text{ and } j = 1, 2\}$ and edge set by $E(D_n) = \{x_{j,i}x_{j,i+1} : j = 1, 2 \text{ and } i \in I\} \cup \{x_{1,i}x_{2,i} : i \in I\}$ with indices taken modulo n. We insert exactly one vertex y (respectively z) into the internal (respectively external) n-sided face of D_n. Suppose that n is even, $n \geq 4$, and consider the graph $\mathbf{D}_n$ with the vertex set $V(\mathbf{D}_n) = V(D_n) \cup \{y, z\}$ and the edge set $E(\mathbf{D}_n) = E(D_n) \cup \{x_{1,2m-1}y : m = 1, 2, ..., \frac{n}{2}\} \cup \{x_{2,2m}z : k = 1, 2, ..., \frac{n}{2}\}$.

The $\mathbf{D}_n$, $n \geq 4$, is the plane graph consisting of $|F(\mathbf{D}_n)| = 2n$ 4-sided faces. If $\mathbf{D}_n$, $n \geq 4$, $n \equiv 0 \pmod 2$, is (a, d)-face antimagic [3] then the feasible values of (a, d) are $(6n + 3, 2)$ or $(4n + 4, 4)$ or $(2n + 5, 6)$ and no other. In [3] it is proved that if n is even, $n \geq 4$, then plane graph $\mathbf{D}_n$ is $(6n + 3, 2)$-face antimagic and $(4n + 4, 4)$-face antimagic. A conjecture that $\mathbf{D}_n$ is $(2n + 5, 6)$-face antimagic is proposed at the end of the paper [3].

The *biprism* B_n, $n \geq 3$, is defined as the Cartesian product $P_3 \times C_n$ of a path on three vertices with a cycle on n vertices, embedded in the plane.

Let us denote the vertex set of B_n by $V(B_n) = \{x_{j,i} : i \in I$ and $j = 1,2,3\}$ and edge set by $E(B_n) = \{x_{j,i}x_{j,i+1} : i \in I$ and $j = 1,2,3\} \cup \{x_{1,i}x_{2,i} : i \in I\} \cup \{x_{2,i}x_{3,i} : i \in I\}$ with indices taken modulo n. We insert exactly one vertex y (z) into the internal (external) n-sided face of B_n. Suppose that n is even, $n \geq 4$, and consider the graph $\mathbf{B}_n$ with the vertex set $V(\mathbf{B}_n) = V(B_n) \cup \{y, z\}$ and the edge set $E(\mathbf{B}_n) = E(B_n) \cup \{x_{1,2m-1}y : m = 1,2,..., \frac{n}{2}\} \cup \{x_{3,2m-1}z : m = 1,2,..., \frac{n}{2}\}$. Then $\mathbf{B}_n$, $n \geq 4$, is the plane graph consisting of $|F(\mathbf{B}_n)| = 3n$ 4-sided faces.

Bača and Miller [6] showed that (a,d)-face antimagic labeling of $\mathbf{B}_n$ do not exist for any values of (a,d) other than $(9n+3,2)$, $(6n+4,4)$, and $(3n+5,6)$. They proved that for n even, $n \geq 4$, the plane graph $\mathbf{B}_n$ has $(9n+3,2)$-face antimagic labeling and $(6n+4,4)$-face antimagic labeling.

For $n \geq 3$ we denote by H_n the plane graph of a convex polytope, which is obtained as a combination of two antiprisms A_n. Let us denote the vertex set of H_n by $V(H_n) = \{x_{j,i} : i \in I$ and $j = 1,2,3\}$ and edge set by $E(H_n) = \{x_{j,i}x_{j,i+1} : i \in I$ and $j = 1,2,3\} \cup \{x_{j,i}x_{j+1,i} : i \in I$ and $j = 1,2\} \cup \{x_{1,i+1}x_{2,i} : i \in I\} \cup \{x_{2,i}x_{3,i+1} : i \in I\}$ with indices taken modulo n. We insert exactly one vertex $y(z)$ into the internal (external) n-sided face of H_n and connect the vertex $y(z)$ with the vertices $x_{1,i}(x_{3,i}), i \in I$. Thus we obtain the plane graph $\mathbf{H}_n$ consisting of $|F(\mathbf{H}_n)| = 6n$ 3-sided faces.

Necessary conditions are given in [4] for (a,d)-face antimagic labeling of $\mathbf{H}_n$: if $\mathbf{H}_n$ is (a,d)-face antimagic then

- for n even: either $d = 1$ and $a = \frac{21n+4}{2}$ or $d = 3$ and $a = \frac{9n+6}{2}$; and
- for n odd: either $d = 2$ and $a = \frac{15n+5}{2}$ or $d = 4$ and $a = \frac{3n+7}{2}$.

The paper [4] describes a $(\frac{21n+4}{2}, 1)$-face antimagic labeling of $\mathbf{H}_n$, $n \equiv 0 \pmod 2$, $n \geq 4$, and poses two conjectures for other feasible values of a and d.

4 Edge-Magic Total Labelings

Edge-magic total labelings were introduced by Kotzig and Rosa [20] as follows. An *edge-magic total labeling* on G will mean a one-to-one map λ from $V(G) \cup E(G)$ onto the integers $1,2,...,v+e$ with the property that, given any edge (x,y),

$$\lambda(x) + \lambda(x,y) + \lambda(y) = k$$

for some constant k. It will be convenient to call $\lambda(x) + \lambda(x,y) + \lambda(y)$ the *edge sum* of (x,y), and k the (constant) *magic sum* of G. A graph is called *edge-magic total* if it admits any edge-magic total labeling.

Ringel and Llado [28] proved that if G has e even and $v+e \equiv 2 \pmod 4$, and every vertex of G has odd degree, then G is not edge-magic total. Kotzig and Rosa [21] showed that no complete graph K_n with $n > 6$ is edge-magic total and neither is K_4, and edge-magic total labelings for K_3, K_5 and K_6, for all feasible values of k, are described in [32].

The cycle C_n is regular of degree 2 and has n edges and the path P_n can be considered as a cycle C_n with an edge deleted. An *n-sun* is a cycle C_n with an additional edge terminating in a vertex of degree 1 attached to each vertex. A *caterpillar* is a graph derived from a path by hanging any number of pendant vertices from vertices of the path. In [20] it is proved that all cycles C_n and caterpillars are edge-magic total. Wallis et al [32] showed that:

- All paths P_n are edge-magic total.
- All suns are edge-magic total.
- The complete bipartite graph $K_{m,n}$ is edge-magic total for any m and n.

It is conjectured that:

- All trees are edge-magic total [20].
- Wheels W_n are edge-magic total whenever $n \not\equiv 3 \pmod 4$ [14].

Enomoto et al [14] checked that :

- All trees with less than 16 vertices are edge-magic total.
- Wheels W_n up to $n = 29$, $n \not\equiv 3 \pmod 4$ are edge-magic total.

5 Vertex-Magic Total Labelings

A one-to-one map λ from $E \cup V$ onto the integers $\{1, 2, \cdots, e+v\}$ is a *vertex-magic total labeling* if there is a constant k so that for every vertex x,

$$\lambda(x) + \sum \lambda(xy) = k$$

where the sum is over all vertices y adjacent to x. Let us call the sum of labels at vertex x the *weight* of the vertex; we require $wt(x) = k$ for all x. The constant k is called the *magic constant* for λ. Vertex-magic total labelings were introduced by MacDougall et al [22]. The edge labels are all distinct (as are all the vertex labels).

In a vertex-magic total labeling, the edges could conceivably receive the e smallest labels or, at the other extreme, the e largest labels, or anything in between. Consequently we have

$$\binom{v+e+1}{2} + \binom{e+1}{2} \leq vk \leq 2\binom{v+e+1}{2} - \binom{v+1}{2}$$

which will give the range of feasible values for the magic constant k. For cycles (and only for cycles) a vertex-magic total labeling is equivalent to an edge-magic total labeling and from previous section it follows

- The n-cycle C_n has a vertex-magic total labeling for any $n \geq 3$.
- The path P_n with n vertices, has a vertex-magic total labeling for any $n \geq 3$.

Complete bipartite graphs provide us with interesting and surprising results. It was shown in [22] that

- If $n > m + 1$ then $K_{m,n}$ has no vertex-magic total labeling.
- There is a vertex-magic total labeling for $K_{m,m}$ for every $m > 1$.

Curiously enough, the proof of this used a magic square construction.
Let G be any graph of order n and size e. We define a *G-sun* to be a graph G^* of order $2n$ formed from G by adjoining n new vertices of degree one to the vertices of G. We have $v^* = |V(G^*)| = 2n$ and $e^* = |E(G^*)| = e + n$. The following result [22] shows that for a vertex-magic total labeling to exist, the number of edges in G must be bounded above by a function of n which is essentially linear:

- Let G be any graph of order n. If G has e edges, then a G-sun G^* has no vertex-magic total labeling whenever

$$e > \frac{-1 + \sqrt{1 + 8n^2}}{2}.$$

It was shown in [22] that there is a vertex-magic total labeling for complete graphs K_n for all odd n. A vertex-magic total labeling of K_{2n}, n odd, is given in [23] using a vertex-magic total labeling of K_n and two mutually orthogonal latin squares of order n. For $n \equiv 0 \pmod 4$ there is stated the following conjecture:

- There is a vertex-magic total labeling for K_n for $n \equiv 0 \pmod 4$.

Next we will turn our attention to the generalized Petersen graph $P(n, m)$ $1 \leq m < \frac{n}{2}$. It was shown in [5] that for n even, $n \geq 4$ and $1 \leq m \leq \frac{n}{2} - 1$, the generalized Petersen graph $P(n, m)$ has edge labeling $f : E \rightarrow \{1, 2, ..., e\}$ which is $(a, 1)$-antimagic.

There trivially exists a vertex labeling with values in the set $\{e+1, e+2, ..., e+v\}$ and the resulting labeling is vertex-magic total.

The plane graph of a convex polytope $\mathbf{H}_n$ was defined in Section 3. A $(\frac{21n+4}{2}, 1)$-face antimagic labeling of $\mathbf{H}_n$, n even, $n \geq 4$, is described in [4]. Clearly, $(\frac{21n+4}{2}, 1)$-face antimagic labeling of $\mathbf{H}_n$, n even, $n \geq 4$, is equivalent to $(\frac{21n+4}{2}, 1)$-antimagic labeling of the dual graph of $\mathbf{H}_n^*$. There trivially exists a vertex labeling of $\mathbf{H}_n^*$, $n \geq 4$, $n \equiv 0 \pmod 2$, with values in the set $\{e+1, e+2, ..., e+v\}$. This proves that

- For n even, $n \geq 4$, the plane graph $\mathbf{H}_n^*$ has a vertex-magic total labeling.

Vertex-magic total labelings for the antiprism A_n, $n \geq 4$, n even, with magic constants $k = 15n + 2$ and $k = 15n + 3$ are given in [26].

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