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A NOTE ON ROMAN DOMINATION OF DIGRAPHS

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Abstract

A vertex subset S of a digraph D is called a dominating set of D if every vertex not in S is adjacent from at least one vertex in S. The domination number of a digraph D, denoted by $\gamma(D)$, is the minimum cardinality of a dominating set of D. A Roman dominating function (RDF) on a digraph D is a function $f : V(D) \to \{0, 1, 2\}$ satisfying the condition that every vertex v with $f(v) = 0$ has an in-neighbor u with $f(u) = 2$. The weight of an RDF f is the value $\omega(f) = \sum_{v \in V(D)} f(v)$. The Roman domination number of a digraph D, denoted by $\gamma_R(D)$, is the minimum weight of an RDF on D. In this paper, for any integer k with $2 \leq k \leq \gamma(D)$, we characterize the digraphs D of order $n \geq 4$ with $\delta^-(D) \geq 1$ for which $\gamma_R(D) = \gamma(D) + k$ holds. We also characterize the digraphs D of order $n \geq k$ with $\gamma_R(D) = k$ for any positive integer k. In addition, we present a Nordhaus-Gaddum bound on the Roman domination number of digraphs.

Keywords: Roman domination number, domination number, digraph, Nordhaus-Gaddum.

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1. Introduction

Domination in graphs, with its many variations, has become an important research topic in graph theory, see, e.g., [10]. Among the variations of domination, so called Roman domination plays an important role in graph theory and its applications. Many results on Roman domination in (undirected) graphs can be found in [1, 4, 5, 7, 12, 13, 16, 18]. Nowadays, also closely related concepts on digraphs have been investigated, for example, signed total Roman domination in digraphs [17] and signed Roman domination in digraphs [15]. By contrast, results on Roman domination in digraphs seldom appear in literature. Our aim in this paper is to study the Roman domination in digraphs.

We would follow Bondy and Murty [2] for graph-theoretical terminology and notation not defined here. Throughout this paper, $D = (V, A)$ denotes a finite digraph with neither loops nor multiple arcs (but pairs of opposite arcs are allowed). For two vertices $u, v \in V(D)$, we use (u, v) to denote the arc with direction from u to v, that is, u is adjacent to v, or equivalently, v is adjacent from u, and we also call v an *out-neighbor* of u and u an *in-neighbor* of v. For a vertex $v \in V(D)$, the *out-neighborhood* and *in-neighborhood* of v, denoted by $N^+(v)$ and $N^-(v)$, are the sets of out-neighbors and in-neighbors of v, respectively. Also, the *closed out-neighborhood* of v is the set $N^+[v] = N^+(v) \cup \{v\}$. In general, for a set $X \subseteq V(D)$, we denote $N^+(X) = \bigcup_{v\in X} N^+(v)$ and $N^+[X] = N^+(X) \cup X$. The *out-degree* and *in-degree* of a vertex $v \in V(D)$ are defined by $d^+(v) = d^+_D(v) = |N^+(v)|$ and $d^-(v) = d^-_D(v) = |N^-(v)|$, respectively. The *maximum out-degree*, *minimum out-degree*, *maximum in-degree* and *minimum in-degree* among the vertices of D are denoted by $\Delta^+(D)$, $\delta^+(D)$, $\Delta^-(D)$ and $\delta^-(D)$, respectively. For a set $X \subseteq V(D)$, the subdigraph induced by X is denoted by $D[X]$. The *complement* $\overline{D}$ of a digraph D is the digraph defined on the vertex set $V(D)$, where $(u, v) \in A(\overline{D})$ if and only if $(u, v) \notin A(D)$. The *complete digraph* K^*_n is the digraph obtained from the complete graph K_n when each edge e of K_n is replaced by two oppositely oriented arcs with the same ends as e.

A vertex subset S of a digraph D is called a *dominating set* of D if $N^+[S] = V(D)$. The *domination number* of a digraph D, denoted by $\gamma(D)$, is the minimum cardinality of a dominating set of D. A dominating set of D of cardinality $\gamma(D)$ is called a $\gamma(D)$-*set*. The domination number of digraphs was introduced by Fu [6], which have been well studied now (see, for example, [3, 8, 9]).

A *Roman dominating function* (RDF) on a digraph D is a function $f : V(D) \to \{0, 1, 2\}$ satisfying the condition that every vertex v with $f(v) = 0$ has an in-neighbor u with $f(u) = 2$. The *weight* of an RDF f is the value $\omega(f) = \sum_{v\in V(D)} f(v)$. The *Roman domination number* of a digraph D, denoted by $\gamma_R(D)$, is the minimum weight of an RDF on D. A $\gamma_R(D)$-*function* is a Roman dominating function on D with weight $\gamma_R(D)$. An RDF f on D can be represented by the ordered partition (V_0, V_1, V_2), where $V_i = \{v \in V(D) : f(v) = i\}$ for $i \in \{0, 1, 2\}$. The Roman domination of a digraph was introduced by Kamaraj and Jakkammal [11].

In this note, we characterize the digraphs D of order $n \geq 4$ with $\delta^-(D) \geq 1$ for which $\gamma_R(D) = \gamma(D) + k$ holds for any integer k with $2 \leq k \leq \gamma(D)$. We also characterize the digraphs D of order $n \geq k$ with $\gamma_R(D) = k$ for any positive integer k. These two results extend some recent results of Sheikholeslami and Volkmann [14]. In addition, we present a Nordhaus-Gaddum inequality for the Roman domination number of digraphs.

2. Main Results

In [14], Sheikholeslami and Volkmann characterized the digraphs D with $\delta^-(D) \geq 1$ for which $\gamma_R(D) = \gamma(D)+k$ holds, where $k \in \{0, 1, 2\}$. Here we would extend their result to an arbitrary integer k with $2 \leq k \leq \gamma(D)$. For this purpose, we first give some needed results.

Proposition 1 [14]**.** *For any digraph D, $\gamma(D) \leq \gamma_R(D) \leq 2\gamma(D)$.*

For any digraph D, it follows from Proposition 1 that if $\gamma_R(D) = \gamma(D) + k$, then $0 \leq k \leq \gamma(D)$.

Proposition 2 [11]**.** *Let $f = (V_0, V_1, V_2)$ be any $\gamma_R(D)$-function on a digraph D. Then*

(a) $\Delta^+(D[V_1]) \leq 1$,

(b) *if $u \in V_1$, then $N^-(u) \cap V_2 = \emptyset$,*

(c) *V_2 is a $\gamma(D[V_0 \cup V_2])$-set.*

Sheikholeslami and Volkmann [14] obtained the exact value of the Roman domination number of directed cycles.

Proposition 3 [14]**.** *If D is a directed cycle of order n, then $\gamma_R(D) = n$.*

Proposition 4 [14]**.** *Let D be a digraph of order n. Then $\gamma_R(D) = \gamma(D)$ if and only if $\Delta^+(D) = 0$.*

Proposition 5 [14]**.** *Let D be a digraph of order $n \geq 2$ with $\delta^-(D) \geq 1$. Then $\gamma_R(D) = \gamma(D) + 1$ if and only if there is a vertex $v \in V(D)$ such that $d^+(v) = n - \gamma(D)$.*

Proposition 6 [14]**.** *Let D be a digraph of order $n \geq 7$ with $\delta^-(D) \geq 1$. Then $\gamma_R(D) = \gamma(D) + 2$ if and only if*

(a) *D does not have a vertex of out-degree $n - \gamma(D)$,*

(b) *either D has a vertex of out-degree $n - \gamma(D) - 1$ or D contains two vertices v, w such that $|N^+[v] \cup N^+[w]| = n - \gamma(D) + 2$.*

In fact, Proposition 6 holds for $n \geq 4$ as the following result shows.

Proposition 7. *Let D be a digraph of order $n \geq 4$ with $\delta^-(D) \geq 1$. Then $\gamma_R(D) = \gamma(D) + 2$ if and only if*

(a) *D does not have a vertex of out-degree $n - \gamma(D)$,*

(b) *either D has a vertex of out-degree $n - \gamma(D) - 1$ or D contains two vertices v, w such that $|N^+[v] \cup N^+[w]| = n - \gamma(D) + 2$.*

Proof. Here we just show the necessity. The proof for the sufficiency is the same as that of Proposition 4 in [14].

Let $\gamma_R(D) = \gamma(D)+2$. Clearly, (a) follows trivially from Proposition 5. Now, let $f = (V_0, V_1, V_2)$ be a $\gamma_R(D)$-function such that $|V_2|$ is maximum. Since $V_1 \cup V_2$ is a dominating set of D, if $|V_1| \leq \gamma(D) - 3$, then

$$\begin{aligned}\gamma(D) \leq& |V_1| + |V_2| = |V_1| + \frac{\gamma_R(D) - |V_1|}{2} = \frac{\gamma_R(D) + |V_1|}{2} \\ \leq& \frac{\gamma_R(D) + \gamma(D) - 3}{2} = \frac{(\gamma(D) + 2) + \gamma(D) - 3}{2} < \gamma(D),\end{aligned}$$

a contradiction. Therefore, we may deduce that one of the following conditions is satisfied.

(i) $|V_1| = \gamma(D) + 2$ and $|V_2| = 0$,

(ii) $|V_1| = \gamma(D)$ and $|V_2| = 1$, and

(iii) $|V_1| = \gamma(D) - 2$ and $|V_2| = 2$.

Suppose first that (i) holds. Clearly, we have $|V_0| = 0$, and then $V_1 = V(D)$. This implies that D is empty (otherwise, there exists at least an arc in D and hence by the choice of $|V_2|$, we have $|V_2| \geq 1$, a contradiction). So in this case, we have $\gamma_R(D) = n \neq n + 2 = \gamma(D) + 2$.

We now suppose that (ii) holds. Let $V_2 = \{u\}$. Since u has no out-neighbors in V_1, by the definition of $\gamma_R(D)$-function, we have $d^+(u) = |V_0| = n-|V_1|-|V_2| = n - \gamma(D) - 1$, as desired.

Finally, suppose that (iii) holds. Let $V_2 = \{v, w\}$. Since neither v nor w has out-neighbors in V_1, by the definition of $\gamma_R(D)$-function, we get $|N^+[v] \cup N^+[w]| = n - |V_1| = n - \gamma(D) + 2$, as required.

This completes the proof. ■

Now we are able to characterize the digraphs D with $\delta^-(D) \geq 1$ for which $\gamma_R(D) = \gamma(D) + k$ holds for any integer k with $2 \leq k \leq \gamma(D)$. It should be mentioned that a similar result for (undirected) graphs has already been given by Xing *et al.* [18].

Theorem 8. *Let D be a digraph of order $n \geq 4$ with $\delta^-(D) \geq 1$ and let k be an integer with $2 \leq k \leq \gamma(D)$. Then $\gamma_R(D) = \gamma(D) + k$ if and only if*

(a) *for any integer s with $1 \leq s \leq k-1$, D does not have a set U_t of t $(1 \leq t \leq s)$ vertices satisfying*
$$|N^+[U_t]| = n - \gamma(D) - s + 2t;$$

(b) *there exists an integer l with $1 \leq l \leq k$ such that D has a set W_l of l vertices satisfying*
$$|N^+[W_l]| = n - \gamma(D) - k + 2l.$$

Proof. We proceed by induction on k. If $k = 2$, then by Proposition 7, the assertion is trivial. Hence, in the following we may assume that $k \geq 3$.

To prove the necessity, suppose that $\gamma_R(D) = \gamma(D) + k$. Let $f = (V_0, V_1, V_2)$ be a $\gamma_R(D)$-function.

First we prove that (a) holds. Suppose, to the contrary, that s_0 $(1 \leq s_0 \leq k-1)$ is the minimum integer such that D has a set U_{t_0} of t_0 $(1 \leq t_0 \leq s_0)$ vertices satisfying $|N^+[U_{t_0}]| = n-\gamma(D)-s_0+2t_0$. If $s_0 = 1$, then $t_0 = 1$. This implies that there exists a vertex $v \in U_{t_0}$ such that $|N^+[v]| = n-\gamma(D)-s_0+2t_0 = n-\gamma(D)+1$ and hence $d^+(v) = n-\gamma(D)$. Thus, by Proposition 5, we have $\gamma_R(D) = \gamma(D)+1$, contradicting our assumption that $\gamma_R(D) = \gamma(D) + k$. Consequently, we have $s_0 \geq 2$, which implies that for any integer s with $1 \leq s \leq s_0 - 1$, D does not have a set U_t of t $(1 \leq t \leq s)$ vertices satisfying $|N^+[U_t]| = n - \gamma(D) - s + 2t$. Since D has a set U_{t_0} of t_0 $(1 \leq t_0 \leq s_0)$ vertices satisfying $|N^+[U_{t_0}]| = n-\gamma(D)-s_0+2t_0$, by the induction hypotheses, we have $\gamma_R(D) = \gamma(D) + s_0$, again contradicting our assumption that $\gamma_R(D) = \gamma(D) + k$. Therefore, (a) holds.

Next we prove that (b) holds. Suppose first that $|V_2| = 0$. By the definition of $\gamma_R(D)$-function, we have $|V_0| = 0$ and hence $V_1 = V(D)$. Also, by condition (a) of Proposition 2, we have $\Delta^+(D) = \Delta^+(D[V_1]) \leq 1$. Now, since $\delta^-(D) \geq 1$,
$$n \leq \sum_{v \in V(D)} \delta^-(D) \leq \sum_{v \in V(D)} d^-(v) = \sum_{v \in V(D)} d^+(v) \leq \sum_{v \in V(D)} \Delta^+(D) \leq n,$$
which implies that $d^+(v) = d^-(v) = 1$ for any vertex $v \in V(D)$, and hence D is a disjoint union of $p \geq 1$ directed cycles. Let $D_i = v_1^i v_2^i \cdots v_{n_i}^i$ be the connected component of D for $i = 1, 2, \ldots, p$. Clearly, $\gamma(D) = \sum_{i=1}^p \lceil n_i/2 \rceil$ and by Proposition 3, we have $\gamma_R(D) = \sum_{i=1}^p n_i = n$. Thus, $k = \gamma_R(D) - \gamma(D) = \sum_{i=1}^p \lfloor n_i/2 \rfloor$. Let $W_l = \bigcup_{i=1}^p \{v_{2j-1}^i : j = 1, 2, \ldots, \lfloor n_i/2 \rfloor\}$, where $l = \sum_{i=1}^p \lfloor n_i/2 \rfloor$. It is easy to see that $|N^+[W_l]| = 2l = n - \gamma(D) - k + 2l$, implying that (b) holds.

We now suppose that $|V_2| \neq 0$. By condition (c) of Proposition 2, V_2 is a $\gamma(D[V_0 \cup V_2])$-set and hence $V_1 \cup V_2$ is a dominating set of D. This implies that $|V_1| + |V_2| \geq \gamma(D)$. Moreover, since $|V_1| + 2|V_2| = \gamma_R(D) = \gamma(D) + k$, $|V_2| \leq k$. Let $|V_2| = l$, where $1 \leq l \leq k$. Then $|V_1| = \gamma(D) + k - 2|V_2| = \gamma(D) + k - 2l$.

By conditions (b) and (c) of Proposition 2, we have $V_1 \cap N^+(V_2) = \emptyset$ and $V_0 \subseteq N^+(V_2)$. Thus, there exists a set $W_l = V_2$ of l $(1 \leq l \leq k)$ vertices such that

$$|N^+[W_l]| = n - |V_1| = n - (\gamma(D) + k - 2l) = n - \gamma(D) - k + 2l,$$

also implying that (b) holds.

To show the sufficiency, suppose that the conditions (a) and (b) in the statement of the theorem hold. We first claim that $\gamma_R(D) \geq \gamma(D)+k$. Suppose, to the contrary, that $\gamma_R(D) = \gamma(D)+m$, where $m \leq k-1$. By the induction hypothesis and condition (b) of the theorem, there exists an integer l with $1 \leq l \leq m \leq k-1$ such that D has a set W_l of l vertices satisfying $|N^+[W_l]| = n - \gamma(D) - m + 2l$, contradicting condition (a). Our claim follows.

Now it remains to show that $\gamma_R(D) \leq \gamma(D) + k$. Let $V_0 = N^+[W_l] - W_l$, $V_1 = V(D) - N^+[W_l]$ and $V_2 = W_l$. It is easy to see that $g = (V_0, V_1, V_2)$ is an RDF on D with weight

$$\begin{aligned} \omega(g) &= |V_1| + 2|V_2| = |V(D)| - |N^+[W_l]| + 2|W_l| \\ &= n - (n - \gamma(D) - k + 2l) + 2l = \gamma(D) + k. \end{aligned}$$

Consequently, we have $\gamma_R(D) \leq \omega(g) = \gamma(D) + k$, as desired.

The proof is completed. ■

Sheikholeslami and Volkmann [14] also characterized the digraphs D with $\gamma_R(D) = k$, where $k \in \{2, 3, 4, 5\}$. Here, we would extend their result to arbitrary positive integer k.

Theorem 9. *For any positive integer k and digraph D of order $n \geq k$, $\gamma_R(D) = k$ if and only if one of the following conditions holds:*

(a) *$n = k$ and $\Delta^+(D) \leq 1$,*

(b) *for any proper subset $X \subset V(D)$ with $1 \leq |X| \leq \lfloor k/2 \rfloor$, $|N^+[X]| \leq n+2|X|-k$. In addition, there exists some proper subset $Y \subset V(D)$ with $1 \leq |Y| \leq \lfloor k/2 \rfloor$ such that $|N^+[Y]| = n + 2|Y| - k$ and $\Delta^+(D[V(D) - N^+[Y]]) \leq 1$.*

Proof. Let $f = (V_0, V_1, V_2)$ be a $\gamma_R(D)$-function. By conditions (b) and (c) of Proposition 2, we have $V_1 \cap N^+[V_2] = \emptyset$ and $V_0 \subset N^+[V_2]$, implying that $V(D) - N^+[V_2] = V_1$ and $|N^+[V_2]| = |V_0| + |V_2|$.

First we prove the sufficiency. Clearly, the assertion holds for $n = k$ and $\Delta^+(D) \leq 1$. Now we consider condition (b). If $\gamma_R(D) = |V_1| + 2|V_2| \leq k - 1$, then $|V_2| \leq (k - 1 - |V_1|)/2 \leq \lfloor k/2 \rfloor$, and by condition (b), we have $|N^+[V_2]| \leq n + 2|V_2| - k$. Thus,

$$k - 1 - 2|V_2| \geq |V_1| = n - (|V_0| + |V_2|) = n - |N^+[V_2]| \geq k - 2|V_2|,$$

a contradiction. Therefore, we obtain $\gamma_R(D) \geq k$. On the other hand, it is easy to see that $f' = (N^+[Y] - Y, V(D) - N^+[Y], Y)$ is an RDF on D with

$$\omega(f') = |V(D) - N^+[Y]| + 2|Y| = n - (n + 2|Y| - k) + 2|Y| = k.$$

Consequently, we have $\gamma_R(D) \leq \omega(f') = k$ and hence $\gamma_R(D) = k$, as desired.

Conversely, suppose that $\gamma_R(D) = k$. If $V_2 = \emptyset$, then by the definition of $\gamma_R(D)$-function, we have $V_0 = \emptyset$ and hence $V_1 = V(D)$. Thus, $k = \gamma_R(D) = |V_1| + 2|V_2| = |V(D)| = n$. Furthermore, by Proposition 2, we have $\Delta^+(D) = \Delta^+(D[V_1]) \leq 1$. Condition (a) follows.

We now assume that $|V_2| \geq 1$. Suppose that there exists some set $X \subset V(D)$ with $1 \leq |X| \leq \lfloor k/2 \rfloor$ such that $|N^+[X]| \geq n + 2|X| + 1 - k$. It is easy to see that $f'' = (N^+[X] - X, V(D) - N^+[X], X)$ is an RDF on D and thus,

$$\begin{aligned}\gamma_R(D) \leq& \omega(f'') = |V(D) - N^+[X]| + 2|X| \\ \leq& n - (n + 2|X| + 1 - k) + 2|X| = k - 1,\end{aligned}$$

a contradiction. Hence, for any set $X \subset V(D)$ with $1 \leq |X| \leq \lfloor k/2 \rfloor$, we have $|N^+[X]| \leq n + 2|X| - k$.

It remains to show that there exists some set $Y \subset V(D)$ with $1 \leq |Y| \leq \lfloor k/2 \rfloor$ such that $|N^+[Y]| = n + 2|Y| - k$ and $\Delta^+(D[V(D) - N^+[Y]]) \leq 1$. Let $Y = V_2$. It is easy to see that $|V_1| + 2|Y| = \gamma(D) = k$ and hence $|Y| = (k - |V_1|)/2 \leq \lfloor k/2 \rfloor$. From the assumptions, we have $|Y| \geq 1$. And as proven above, we get $|N^+[Y]| \leq n + 2|Y| - k$. Thus, it follows that

$$|V_1| = n - (|V_0| + |Y|) = n - |N^+[Y]| \geq k - 2|Y| = |V_1|,$$

which implies that $|N^+[Y]| = n + 2|Y| - k$. Consequently, by condition (a) of Proposition 2, we have $\Delta^+(D[V(D) - N^+[Y]]) = \Delta^+(D[V_1]) \leq 1$.

This completes the proof. ∎

Finally, we give a Nordhaus-Gaddum bound on the Roman domination number of digraphs. First we need a result of Sheikholeslami and Volkmann [14].

Proposition 10 [14]**.** *If D is a digraph of order n, then*

$$\gamma_R(D) \leq n - \Delta^+(D) + 1.$$

Theorem 11. *If D is a digraph of order $n \geq 3$, then*

$$\gamma_R(D) + \gamma_R(\overline{D}) \leq n + 3,$$

and this bound is sharp.

Proof. Noting that $d^+_D(v) + d^+_{\overline{D}}(v) = n - 1$ holds for any vertex $v \in V(D)$, we have $\Delta^+(\overline{D}) = n - 1 - \delta^+(D)$. Now by Proposition 10, we have

$$\begin{aligned}\gamma_R(D) + \gamma_R(\overline{D}) &\leq (n - \Delta^+(D) + 1) + (n - \Delta^+(\overline{D}) + 1)\\ &= n - \Delta^+(D) + \delta^+(D) + 3 \leq n + 3,\end{aligned}$$

as desired.

To see the sharpness of this bound, consider the digraph D which is obtained from the complete digraph K^*_{n-1} by adding a new vertex u and $n-2$ new arcs from u to any vertex in $V(K^*_{n-1}) \setminus \{v\}$, where v is a vertex of K^*_{n-1}. It is easy to see that $(V(D) \setminus \{u, v\}, \{u\}, \{v\})$ and $(\{v\}, V(D) \setminus \{u, v\}, \{u\})$ are a $\gamma_R(D)$-function and a $\gamma_R(\overline{D})$-function, respectively, and hence

$$\gamma_R(D) + \gamma_R(\overline{D}) = (1 + 2) + (n - 2 + 2) = n + 3,$$

which implies that the bound in this theorem is sharp, completing the proof. ■

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