Discussiones Mathematicae
Graph Theory 38 (2018) 397–404
doi:10.7151/dmgt.2009

A NOTE ON THE RAMSEY NUMBER OF EVEN WHEELS VERSUS STARS

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Abstract

For two graphs G_1 and G_2, the *Ramsey number* $R(G_1, G_2)$ is the smallest integer N, such that for any graph on N vertices, either G contains G_1 or $\overline{G}$ contains G_2. Let S_n be a star of order n and W_m be a wheel of order $m + 1$. In this paper, we will show $R(W_n, S_n) \leq 5n/2 - 1$, where $n \geq 6$ is even. Also, by using this theorem, we conclude that $R(W_n, S_n) = 5n/2 - 2$ or $5n/2 - 1$, for $n \geq 6$ and even. Finally, we prove that for sufficiently large even n we have $R(W_n, S_n) = 5n/2 - 2$.

Keywords: Ramsey number, star, wheel, weakly pancyclic.

2010 Mathematics Subject Classification: 05C55, 05D10.

1. Introduction and Background

Let $G = (V, E)$ denote a finite simple graph on the vertex set V and the edge set E. For the terms undefined here you can see [2]. The subgraph of G *induced* by $S \subseteq V$, $G[S]$, is a graph with vertex set S and two vertices of S are adjacent in $G[S]$ if and only if they are adjacent in G. The complement of a graph G is denoted by $\overline{G}$. For a vertex $v \in V(G)$, we denote the set of all neighbors of v by $N_G(v)$ (or $N(v)$). The degree of a vertex v in a graph G, denoted by $deg_G(v)$ (or $deg(v)$), is the size of the set $N(v)$. The minimum degree, maximum degree and clique number of G are denoted by $\delta(G)$, $\Delta(G)$ and $\omega(G)$, respectively. The *girth* of graph G, $g(G)$, is the length of shortest cycle. Also, the *circumference* of graph

G is the length of longest cycle in G and is denoted by $c(G)$. A graph G of order n is called *Hamiltonian*, *pancyclic* and *weakly pancyclic* if it contains C_n, cycles of every length between 3 and n, and cycles of every length l with $g(G) \leq l \leq c(G)$, respectively. We say that G is a *join* graph if G is the complete union of two graphs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$. In other words, $V = V_1 \cup V_2$ and $E = E_1 \cup E_2 \cup \{uv : u \in V_1, v \in V_2\}$. If G is the join graph of G_1 and G_2, we shall write $G = G_1 + G_2$. A *wheel* W_m is a graph on $m+1$ vertices obtained from C_m by adding one vertex which is called the *hub* and joining each vertex of C_m to the hub with the edges called the *rim* of the wheel. In other words, $W_m = C_m + K_1$. A *star* S_n is the complete bipartite graph $K_{1,n-1}$.

For two graphs G_1 and G_2, the *Ramsey number* $R(G_1, G_2)$ is the smallest positive integer N such that for every graph G on N vertices, G contains G_1 as a subgraph or the complement of G contains G_2 as a subgraph. Chvátal and Harary in [4] proved the following lower bound for Ramsey numbers:

$$R(G, H) \geq (\chi(G) - 1) \cdot (l(H) - 1) + 1,$$

where $l(H)$ is the number of vertices in the largest connected component of H and $\chi(G)$ is the chromatic number of G.

In this note, we consider the Ramsey number for stars versus wheels. The Harary lower bound for $R(W_m, S_n)$ is $3n - 2$ or $2n - 1$, where m is odd or even, respectively. There are many results about this Ramsey number when m is odd. Chen *et al.* in the year 2004 proved that if $m \leq n + 1$ and m is odd, then $R(W_m, S_n) = 3n - 2$ which is the Harary lower bound (see [3]). Also, one year later, Hasmawati *et al.* extended this bound for m. They showed that $R(W_m, S_n) = 3n - 2$, for the case $m \leq 2n - 3$ in [8]. But, one can see in [7], if $n \geq 2$ and $m \geq 2n - 2$, then $R(W_m, S_n) = n + m - 1$, where m is odd.

Also, one can find many results about $R(W_m, S_n)$ when m is even. Surahmat and Baskoro in [12] verified this Ramsey number for the case $m = 4$ in 2001. They proved that $R(W_4, S_n) = 2n - 1$ if $n \geq 3$ and odd, and $R(W_4, S_n) = 2n + 1$ if $n \geq 4$ and even. Korolova in [9] found a lower bound which improved the Harary lower bound. In fact Korolova proved that $R(W_m, S_n) \geq 2n + 1$ for all $n \geq m \geq 6$ and m even. Also, Chen *et al.* in [3] showed that this lower bound is sharp for $m = 6$. In other words, they proved that $R(W_6, S_n) = 2n + 1$. It was proved in [14] that $R(W_8, S_n) = 2n + 2$ for $n \geq 6$ and even in the year 2008. Also, one year later, the exact value of $R(W_8, S_n)$ for odd n was determined. In fact, it was shown in [13] that $R(W_8, S_n) = 2n + 1$ for $n \geq 5$ and odd in the year 2009.

Li and Schiermeyer in [10] indicated two following theorems in which they obtained a new lower bound and showed that for some cases this bound is sharp.

Theorem 1 [10]**.** *If* $6 \leq m \leq 2n-4$ *and* m *is even, then*

$$R(W_m, S_n) \geq \begin{cases} 2n + m/2 - 3 & \text{if } n \text{ is odd and } m/2 \text{ is even,} \\ 2n + m/2 - 2 & \text{otherwise.} \end{cases}$$

Theorem 2 [10]**.** *If* $n+1 \leq m \leq 2n-4$ *and* m *is even, then*

$$R(W_m, S_n) = \begin{cases} 2n + m/2 - 3 & \text{if } n \text{ is odd and } m/2 \text{ is even,} \\ 2n + m/2 - 2 & \text{otherwise.} \end{cases}$$

But for some cases, $R(W_m, S_n)$, where m is even, is still open. One of these cases is when $m = n$. It was shown in [9] that $R(W_n, S_n) \leq 3n - 3$ when n is even. In this paper, we will improve this upper bound and prove the following.

Theorem 3. $R(W_n, S_n) \leq 5n/2 - 1$, *where* $n \geq 6$ *is even.*

Finally, we have the following theorem.

Theorem 4. *For sufficiently large even* n *we have* $R(W_n, S_n) \leq 5n/2 - 2$.

2. Preliminary Lemmas and Theorems

To prove Theorem 3, we need some theorems and lemmas.

Lemma 5 (Brandt *et al.* [1])**.** *Every non-bipartite graph* G *of order* n *with* $\delta(G) \geq (n+2)/3$ *is weakly pancyclic with* $g(G) = 3$ *or* $g(G) = 4$.

Lemma 6 (Dirac [5])**.** *Let* G *be a* 2*-connected graph of order* $n \geq 3$ *with* $\delta(G) = \delta$. *Then* $c(G) \geq \min\{2\delta, n\}$.

Theorem 7 (Faudree and Schelp [6], Rosta [11])**.**

$$R(C_n, C_m) = \begin{cases} 2n - 1 & \text{for } 3 \leq m \leq n,\ m \text{ odd } (n,m) \neq (3,3), \\ n + m/2 - 1 & \text{for } 4 \leq m \leq n,\ m, n \text{ even } (n,m) \neq (4,4), \\ \max\{n + m/2 - 1, 2m - 1\} & \text{for } 4 \leq m < n,\ m \text{ even and } n \text{ odd.} \end{cases}$$

Lemma 8 [2]**.** *Let* G *be a bipartite graph of order* n (n *even*) *with bipartition* (X, Y) *and* $|X| = |Y| = n/2$. *If for all distinct nonadjacent vertices* $u \in X$ *and* $v \in Y$, *we have* $deg(u) + deg(v) > n/2$, *then* G *is Hamiltonian.*

3. Proof of Theorem 3

From now on, let G be a graph of order $N = 5n/2 - 1$, where $n \geq 6$ and n is even, such that neither G contains W_n nor its complement, $\overline{G}$, contains S_n. Also, for every vertex $t \in V(G)$ consider $H_t = G[N(t)]$ and $\overline{H_t} = \overline{G}[N(t)]$. Since $\overline{G}$ has no S_n, $deg_{\overline{G}}(v) \leq n-2$, for each vertex $v \in V(G)$. Thus, $\delta(G) \geq 3n/2$. In the middle of the proof, we sometimes interrupt it and have some lemmas.

Let $v_0 \in V(G)$ be an arbitrary vertex. There exists a $k \in \{0, 1, 2, \ldots, n-2\}$ such that $deg_G(v_0) = 3n/2 + k$, since $\delta(G) \geq 3n/2$. Thus, the order of $H_{v_0} = G[N(v_0)]$ is $3n/2 + k$. By the second part of Theorem 7, we have $|V(H_{v_0})| = 3n/2 + k \geq R(C_n, C_s)$, where $s = 2l$, and l is an integer such that $4 \leq 2l \leq n + k + 1$. (Note that in Theorem 3 we have $n \geq 6$, so the case $(n, s) = (4, 4)$ does not occur for $R(C_n, C_s)$ in Theorem 7). Thus, either H_{v_0} contains C_n or $\overline{H}_{v_0}$ contains C_s. But if H_{v_0} contains C_n, then G contains W_n, which is a contradiction. Hence we have the following corollary.

Corollary 9. *Let $v \in V(G)$ and k be an element in the set $\{0, 1, \ldots, n-2\}$ such that $|V(H_v)| = 3n/2 + k$. Then $\overline{H}_v$ contains C_{2l} for all integers l such that $4 \leq 2l \leq n + k + 1$.*

Proposition 10. $\omega(\overline{G}) \leq n-2$ *and* $\omega(G) \leq n-1$.

Proof. It is clear that $\omega\left(\overline{G}\right) \leq n-1$, since $\Delta\left(\overline{G}\right) \leq n-2$. Suppose $\omega\left(\overline{G}\right) = n-1$ and $T = \{v_1, \ldots, v_{n-1}\}$ is a clique in $\overline{G}$. For any $v \in V - T$, $N_{\overline{G}}(v) \cap T = \emptyset$, otherwise $\overline{G}[T \cup \{v\}]$ contains S_n. Now consider $v \in V - T$ and let k be an element in the set $\{0, 1, \ldots, n-2\}$ such that $|V(H_v)| = 3n/2 + k$. Since $N_{\overline{G}}(v) \cap T = \emptyset$, the set $V(H_v)$ contains the set T. It means that $\overline{G}[T]$ is a connected component of $\overline{H}_v$ in the graph $\overline{G}$. On the other hand, by Corollary 9, $\overline{H}_v$ contains a cycle C of length $2l$, where $l = \lfloor (n+k+1)/2 \rfloor$. Note that $C \not\subseteq T$, since $2l > n-1$. Thus, $C \subseteq \overline{H}_v - T$. But $\overline{H}_v - T$ has $n/2 + k + 1$ vertices, which is less than $2l$, a contradiction. Hence $\omega\left(\overline{G}\right) \leq n-2$. For the second part, assume to the contrary, G contains K_n and $H = G[V - K_n]$. Then $|N_G(v) \cap K_n| \geq 2$ for all $v \in V(H)$, otherwise $deg_{\overline{G}}(v) \geq n-1$, which is a contradiction. If $|N_G(v) \cap K_n| = 2$ for all $v \in V(H)$, then $H = K_{3n/2-1}$, since $\delta(G) \geq 3n/2$. But $K_{3n/2-1}$ contains W_n, a contradiction. So, there is a vertex $u \in V(H)$ such that $|N_G(u) \cap K_n| \geq 3$. But $\{u\} \cup K_n$ contains W_n, which is a contradiction. Thus $\omega(G) \leq n-1$. ■

We can divide the proof into some cases and subcases.

Case 1. There is a vertex $v \in V(G)$ for which H_v is bipartite. Let H_v be a bipartite graph, with bipartition (X_v, Y_v), of order $3n/2 + k$ such that $k \in \{0, 1, \ldots, n-2\}$. Without loss of generality, suppose that $|X_v| \leq |Y_v|$. Thus, by Proposition 10, we have $n/2 + k + 2 \leq |X_v| \leq 3n/4 + k/2$ and $3n/4 + k/2 \leq |Y_v| \leq n-2$.

Let $|X_v| = n/2 + s$, where s is an integer such that $k + 2 \leq s \leq n/4 + k/2$. Then $|Y_v| = n + k - s$. Since $\Delta(\overline{G}) \leq n-2$ and $|V(H_v)| = 3n/2 + k$, we conclude $\delta(H_v) \geq n/2 + k + 1$. Let X'_v and Y'_v be obtained from X_v and Y_v by deleting s and $n/2+k-s$ arbitrary vertices, respectively, and let $H'_v = (X'_v, Y'_v)$. Thus, $|X'_v| = |Y'_v| = n/2$ and $\delta(X'_v) \geq s+1$ and $\delta(Y'_v) \geq n/2+k+1-s$ in H'_v. Hence for each two vertices $u_1 \in X'_v$ and $u_2 \in Y'_v$, we have $deg(u_1) + deg(u_2) \geq n/2+k+2$ and by Lemma 8, H'_v contains C_n. It means that G contains W_n, which is a contradiction.

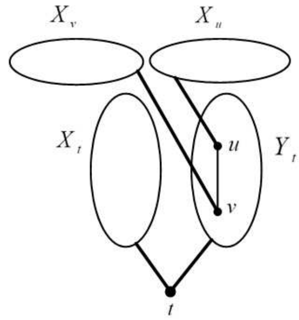

Figure 1. The disjoint sets X_t, Y_t, X_u and X_v.

Case 2. For every vertex $t \in V(G)$, H_t is non-bipartite.

Subcase 2.1. Suppose H_t is disconnected for all $t \in V(G)$. Let $t \in V(G)$ be an arbitrary vertex and $|V(H_t)| = 3n/2+k$, where $k \in \{0,1,2,\ldots,n-2\}$. We show that H_t has exactly two connected components. Suppose to the contrary, H_1, H_2 and H_3 are three connected components of H_t. Since $\delta(H_t) \geq n/2+k+1$, we conclude $\delta(H_i) \geq n/2+k+1$ for $i = 1,2,3$. Hence $|V(H_t)| > 3n/2+k$, which is a contradiction. Now, let X_t, Y_t be the set of vertices of two components of H_t. Assume that $|X_t| \leq |Y_t|$. We choose two adjacent vertices u and v in Y_t, since $\delta(H_t) \geq n/2+k+1$. Let $|V(H_u)| = 3n/2+k'$ and $|V(H_v)| = 3n/2+k''$, where $k', k'' \in \{0,1,2,\ldots,n-2\}$. Also, let X_u, Y_u and X_v, Y_v be the sets of vertices of two components of H_u and H_v, respectively. Since H_t and H_u are disconnected, X_u or Y_u is disjoint from X_t and Y_t. To see this, with no loss of generality, suppose that v is contained in Y_u. Thus, $t \in Y_u$ and hence $X_u \cap Y_t = X_u \cap X_t = \emptyset$. Similarly, X_v or Y_v, say X_v, is disjoint from X_t and Y_t. Thus, we have $Y_t \cap X_u = Y_t \cap X_v = X_t \cap X_u = X_t \cap X_v = \emptyset$. Also, $X_u \cap X_v = \emptyset$; otherwise if $l \in X_u \cap X_v$, then l is adjacent to both u and v. But $u \in Y_v$ implies that $l \in Y_v$. It means, $X_v \cap Y_v \neq \emptyset$ which is a contradiction (see Figure 1). Thus, $X_u \cap X_v = \emptyset$. Hence $|V(G)| \geq |V(H_t)| + |X_u| + |X_v|$ which means $|V(G)| \geq (3n/2+k) + (n/2+k'+2) + (n/2+k''+2) > 5n/2-1$, which is a contradiction.

Subcase 2.2. Suppose H_t is connected for some $t \in V(G)$. Assume that there exists a vertex $u \in V(G)$ for which H_u is 2-connected and $|V(H_u)| = 3n/2+k$ for some $k \in \{0,1,2,\ldots,n-2\}$. Thus, $\delta(H_u) \geq n/2+k+1 \geq (3n/2+k+2)/3$

and by Lemma 5, H_u is weakly pancyclic with $g(H_u) = 3$ or $g(H_u) = 4$. Also, by Lemma 6, $c(H_u) \geq \min\{2\delta(H_u), 3n/2 + k\}$. Hence $c(H_u) \geq n$ which implies that H_u contains C_n, a contradiction.

Now, assume each connected H_t contains a cut-vertex. Let u be a cut-vertex of H_t and $|V(H_t)| = 3n/2 + k$. We show that $H_t - u$ has exactly two connected components. Suppose to the contrary, H_1, H_2 and H_3 are three connected components of $H_t - u$. Since $\delta(H_t) \geq n/2 + k + 1$, $\delta(H_i) \geq n/2 + k$ for $i = 1, 2, 3$. Hence $|V(H_t)| > 3n/2 + k$, which is a contradiction. Now, let s_1 be a cut-vertex of H_t and X_t, Y_t be the sets of vertices of two components of $H_t - s_1$. Assume that $|X_t| \leq |Y_t|$. We choose two adjacent vertices u and v in Y_t, since $\delta(H_t) \geq n/2 + k + 1$. With no loss of generality, suppose that v is contained in Y_u and u is contained in Y_v. Thus, $t \in Y_u \cap Y_v$. Let s_2 and s_3 be the cut-vertices of H_u and H_v, respectively (if any of these cut-vertices did not exist, for instance s_1, then the corresponding subgraph, H_t, is disconnected and the procedure is the same as in Subcase 2.1) and $|V(H_u)| = 3n/2 + k'$ and $|V(H_v)| = 3n/2 + k''$, where $k', k'' \in \{0, 1, 2, \ldots, n-2\}$. Also, let X_u, Y_u and X_v, Y_v be the sets of vertices of two components of $H_u - s_2$ and $H_v - s_3$, respectively. Since $H_t - s_1$, $H_u - s_2$ and $H_v - s_3$ are disconnected, with the same statement of Subcase 2.1 and without loss of generality, we have $Y_t \cap X_u = Y_t \cap X_v = X_t \cap X_u = X_t \cap X_v = X_u \cap X_v = \emptyset$ (see Figure 1). Hence by the fact that $s_1 \notin X_u \cup X_v$ (since otherwise, if for instance $s_1 \in X_u$, then $t \in X_u$ but $t \in Y_u$, a contradiction) we have $|V(G)| \geq |V(H_t - s_1)| + |X_u| + |X_v| + |\{s_1\}|$ which means $|V(G)| \geq (3n/2 + k - 1) + (n/2 + k' + 1) + (n/2 + k'' + 1) + 1 > 5n/2 - 1$, which is a contradiction, and this completes the proof.

Now, by Theorems 1 and 3, the following corollary is obvious.

Corollary 11. *For $n \geq 6$ and even, we have $R(W_n, S_n) = 5n/2 - 2$ or $5n/2 - 1$.*

4. PROOF OF THEOREM 4

We say *n is sufficiently large* if there is a graph G of order n such that $\delta(G) \geq n/4 + 250$. In this section, we prove that for sufficiently large even n we have $R(W_n, S_n) = 5n/2 - 2$. In order to prove this, we use following lemma.

Lemma 12 [1]**.** *If G is a 2-connected non-bipartite graph of sufficiently large order n with $\delta(G) > 2n/7$, then G is weakly pancyclic.*

Let G be a graph of order $N = 5n/2 - 2$, where n is sufficiently large and even such that neither G contains W_n nor its complement, $\overline{G}$, contains S_n. We define H_t for each $t \in V(G)$ similarly as in the proof of Theorem 3. Since $\overline{G}$ has no S_n, $\delta(G) \geq 3n/2 - 1$. Let $v_0 \in V(G)$ be an arbitrary vertex. There exists a

$k \in \{-1, 0, 1, \ldots, n-3\}$ such that $deg_G(v_0) = 3n/2 + k$, since $\delta(G) \geq 3n/2 - 1$. (Here, k is the element of the set $\{-1, 0, 1, \ldots, n-3\}$. This is the only difference of this proof with the proof of Theorem 3). It is easy to check that Corollary 9 and Proposition 10 are true here.

We can divide the proof into some cases and subcases.

Case 1. There is a vertex $v \in V(G)$ for which H_v is bipartite. Let H_v be a bipartite graph with bipartition (X_v, Y_v) of order $3n/2 + k$ such that $k \in \{-1, 0, 1, \ldots, n-3\}$. The sketch of the proof is the same as in Case 1 of the proof of Theorem 3.

Case 2. For every vertex $t \in V(G)$, H_t is non-bipartite.

Subcase 2.1. Suppose H_t is disconnected for all $t \in V(G)$. In Subcase 2.1 of Theorem 3, let k, k' and k'' be in the set $\{-1, 0, \ldots, n-3\}$. The rest of the proof is the same. Finally, we obtain $|V(G)| \geq (3n/2 + k) + (n/2 + k' + 2) + (n/2 + k'' + 2) > 5n/2 - 2$, which is a contradiction.

Subcase 2.2. Suppose H_t is connected for some $t \in V(G)$. Assume that there exists a vertex $u \in V(G)$ for which H_u is 2-connected and $|V(H_u)| = 3n/2 + k$ for some $k \in \{-1, 0, 1, \ldots, n-3\}$. Thus, $\delta(H_u) \geq n/2 + k + 1 > 2(3n/2 + k)/7$ and by Lemma 12, H_u is weakly pancyclic. Also, by Lemma 6, $c(H_u) \geq \min\{2\delta(H_u), 3n/2 + k\}$. Hence $c(H_u) \geq n$ which implies that H_u contains C_n, a contradiction.

Now, assume each connected H_t contains a cut-vertex. In Subcase 2.2 of Theorem 3, let k, k' and k'' be in the set $\{-1, 0, \ldots, n-3\}$. The rest of the proof is the same. Finally, we obtain $|V(G)| \geq (3n/2 + k - 1) + (n/2 + k' + 1) + (n/2 + k'' + 1) + 1 > 5n/2 - 2$, which is a contradiction, and this completes the proof.

Now, by Theorems 1 and 4, the following corollary is obvious.

Corollary 13. *For sufficiently large even n, we have $R(W_n, S_n) = 5n/2 - 2$.*

Acknowledgement

We thank the reviewers for their thorough review and highly appreciate comments and suggestions, which significantly contributed to improving the quality of the publication.

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Received 14 June 2016
Revised 1 December 2016
Accepted 1 December 2016