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THE EXISTENCE OF $P_{\geq 3}$-FACTOR COVERED GRAPHS

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Abstract

A spanning subgraph F of a graph G is called a $P_{\geq 3}$-factor of G if every component of F is a path of order at least 3. A graph G is called a $P_{\geq 3}$-factor covered graph if G has a $P_{\geq 3}$-factor including e for any $e \in E(G)$. In this paper, we obtain three sufficient conditions for graphs to be $P_{\geq 3}$-factor covered graphs. Furthermore, it is shown that the results are sharp.

Keywords: $P_{\geq 3}$-factor, $P_{\geq 3}$-factor covered graph, toughness, isolated toughness, regular graph.

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1. Introduction

The graphs considered in this paper are finite, undirected and simple. We denote by $G = (V(G), E(G))$ a graph, where $V(G)$ and $E(G)$ denote its vertex set and edge set respectively. For $x \in V(G)$, the degree of x in G is denoted by $d_G(x)$. For $S \subseteq V(G)$, we use $G - S$ to denote the subgraph obtained from G by deleting vertices in S together with edges incident to vertices in S. A set $S \subseteq V(G)$ is said to be independent if no two vertices in S are adjacent to each other. The number of isolated vertices of a graph G is denoted by $i(G)$. We use $\omega(G)$ to denote the number of components of a graph G. Other basic graph-theoretic terminologies can be found in [4].

A factor of a graph is a spanning subgraph of the graph. Especially, a (g, f)-factor of a graph G is defined as a spanning subgraph F such that $g(x) \leq d_F(x) \leq f(x)$ for each $x \in V(G)$, where $g(x)$ and $f(x)$ are two nonnegative integer-valued functions defined on $V(G)$ with $g(x) \leq f(x)$ for any $x \in V(G)$. If $g(x) = f(x) = k$ for any $x \in V(G)$, then a (g, f)-factor of G is called a k-factor. A 1-factor is also called a perfect matching. Since all of these notions concern the degree of vertices, they are often defined as *degree factors*. Degree factors in graphs attract a great deal of attentions [2, 7, 11, 13, 15, 16, 17].

On the other hand, when we focus on components of a factor, we lead to the notion of *component factors*. For a set $\mathcal{H}$ of connected graphs, an $\mathcal{H}$-factor of a graph G is a spanning subgraph F of G if every component of F is isomorphic to an element of $\mathcal{H}$. Especially, if each component of F is a path, then F is said to be a path-factor. Apparently, a 1-factor is a P_2-factor. A $P_{\geq k}$-factor means a path-factor in which every component path has at least k vertices, where $k \geq 2$. A graph G is defined as a $P_{\geq k}$-factor covered graph if G admits a $P_{\geq k}$-factor including e for any $e \in E(G)$.

Egawa, Fujita and Ota [6] studied the existence of $K_{1,3}$-factors in graphs. Kano, Lu and Yu [10] presented a sufficient condition for graphs to have $\{K_{1,2}, K_{1,3}, K_5\}$-factors. Kano and Saito [12] obtained a result on the existence of a $\{K_{1,l} : m \leq l \leq 2m\}$-factor and conjectured that a graph G satisfying $i(G-S) \leq \frac{|S|}{m}$ for each $S \subseteq V(G)$ actually contains a $(\{K_{1,l} : m \leq l \leq 2m-1\} \cup \{K_{2m+1}\})$-factor, where $m \geq 2$ is an integer. Zhang, Yan and Kano [18] proved that the conjecture above is true. Akiyama, Avis and Era [1] showed a necessary and sufficient condition for a graph to have a $P_{\geq 2}$-factor. Bazgan, Benhamdine, Li and Woźniak [3] posed a toughness condition for the existence of a $P_{\geq 3}$-factor in a graph. Kaneko [8] obtained a criterion for a graph to have a $P_{\geq 3}$-factor. A simpler proof was posed by Kano, Katona and Király [9]. Zhang and Zhou [19] gave a characterization for $P_{\geq 3}$-factor covered graphs.

A graph R is said to be factor-critical if $R - x$ includes a 1-factor (P_2-factor) for any $x \in V(R)$. A graph H is said to be a sun if $H = K_1$, $H = K_2$ or H is the corona of a factor-critical graph R with at least three vertices, i.e., H is obtained from R by adding a new vertex $w = w(v)$ together with a new edge vw for any $v \in V(R)$. A sun with at least six vertices is said to be a big sun. We use $sun(G)$ to denote the number of sun components of G.

Kaneko [8] presented a criterion for a graph to have a $P_{\geq 3}$-factor.

Theorem 1 (Kaneko [8])**.** *A graph G contains a $P_{\geq 3}$-factor if and only if $sun(G-S) \leq 2|S|$ for any subset S of $V(G)$.*

Zhang and Zhou [19] extended Theorem 1 to $P_{\geq 3}$-factor covered graphs and obtained a characterization for $P_{\geq 3}$-factor covered graphs.

Theorem 2 (Zhang and Zhou [19]). *Let G be a connected graph. Then G is a $P_{\geq 3}$-factor covered graph if and only if $sun(G-S) \leq 2|S| - \varepsilon(S)$ for any subset S of $V(G)$, where $\varepsilon(S)$ is defined by*

$$\varepsilon(S) = \begin{cases} 2, & if\ S \neq \emptyset \text{ and } S \text{ is not an independent set}, \\ 1, & if\ S \neq \emptyset,\ S \text{ is an independent set and there exists a} \\ & \text{non-sun component } of\ G-S, \\ 0, & otherwise. \end{cases}$$

In this paper, we proceed to investigate $P_{\geq 3}$-factor covered graphs and obtain some sufficient conditions for the existence of $P_{\geq 3}$-factor covered graphs. Our main results will be shown in Sections 2, 3 and 4, respectively.

2. Toughness and $P_{\geq 3}$-Factor Covered Graphs

The toughness $t(G)$ of a graph G was first defined by Chvátal in [5] as follows.

$$t(G) = \min\left\{\frac{|S|}{\omega(G-S)} : S \subseteq V(G), \omega(G-S) \geq 2\right\},$$

if G is not complete; otherwise, $t(G) = +\infty$. Bazgan, Benhamdine, Li and Woźniak [3] showed a toughness condition for the existence of a $P_{\geq 3}$-factor in a graph.

Theorem 3 (Bazgan, Benhamdine, Li and Woźniak [3]). *Let G be a graph with at least three vertices. If $t(G) \geq 1$, then G includes a $P_{\geq 3}$-factor.*

The following theorem is a generalization and improvement of Theorem 3.

Theorem 4. *Let G be a connected graph with at least three vertices. If $t(G) > \frac{2}{3}$, then G is a $P_{\geq 3}$-factor covered graph.*

Remark 5. The result in Theorem 4 is sharp. To see this, we construct a graph $G = K_2 \vee (H_1 \cup H_2 \cup H_3)$, where H_i is a sun for $1 \leq i \leq 3$. Set $S = V(K_2)$. It is easy to see that $sun(G-S) = \omega(G-S) = 3$ and $t(G) = \min\left\{\frac{|X|}{\omega(G-X)} : X \subseteq V(G), \omega(G-X) \geq 2\right\} = \frac{|S|}{\omega(G-S)} = \frac{2}{3}$. Note that $\varepsilon(S) = 2$. Hence, we obtain

$$sun(G-S) = 3 > 2 = 2|S| - \varepsilon(S).$$

In terms of Theorem 2, G is not a $P_{\geq 3}$-factor covered graph.

***Proof of Theorem* 4.** If G is a complete graph, obviously G is a $P_{\geq 3}$-factor covered graph as $|V(G)| \geq 3$. In the following, we assume that G is not a complete graph. Suppose that G satisfies the conditions of in Theorem 4, but it is not a $P_{\geq 3}$-factor covered graph. Then by Theorem 2, there exists a subset S of $V(G)$ such that

$$sun(G - S) > 2|S| - \varepsilon(S). \tag{1}$$

We shall consider three cases by the value of $|S|$ and derive a contradiction in each case.

Case 1. $|S| = 0$. In this case, we have $\varepsilon(S) = 0$. In terms of (1), we obtain

$$sun(G) > 0.$$

According to the integrity of $sun(G)$, we have

$$sun(G) \geq 1. \tag{2}$$

On the other hand, since G is connected, we obtain

$$sun(G) \leq \omega(G) = 1.$$

Combining this with (2), we have

$$sun(G) = \omega(G) = 1. \tag{3}$$

According to (3), $|V(G)| \geq 3$ and the definition of sun, it is easy to see that G is a big sun. We denote by R the factor-critical subgraph of G. For any $u \in V(R)$, we write $X = \{u\}$. Clearly, $\omega(G - X) \geq 2$. In terms of the definition of $t(G)$, we obtain

$$t(G) \leq \frac{|X|}{\omega(G - X)} \leq \frac{1}{2},$$

which contradicts $t(G) > \frac{2}{3}$.

Case 2. $|S| = 1$. In this case, we obtain $\varepsilon(S) \leq 1$. According to (1), we have

$$sun(G - S) > 2|S| - \varepsilon(S) \geq 2 - 1 = 1.$$

In terms of the integrity of $sun(G - S)$, we obtain

$$sun(G - S) \geq 2.$$

Note that $\omega(G - S) \geq sun(G - S)$. Combining this with $t(G) > \frac{2}{3}$, we have

$$\frac{2}{3} < t(G) \leq \frac{|S|}{\omega(G - S)} \leq \frac{|S|}{sun(G - S)} \leq \frac{1}{2},$$

which is a contradiction.

Case 3. $|S| \geq 2$. Note that $\varepsilon(S) \leq 2$. It follows from (1) that

$$sun(G-S) \geq 2|S| - \varepsilon(S) + 1 \geq 2|S| - 1,$$

which implies

$$|S| \leq \frac{sun(G-S)+1}{2} \tag{4}$$

and

$$sun(G-S) \geq 3. \tag{5}$$

In terms of (4), (5), $\omega(G-S) \geq sun(G-S)$ and the definition of $t(G)$, we obtain

$$\begin{aligned} t(G) &\leq \frac{|S|}{\omega(G-S)} \leq \frac{|S|}{sun(G-S)} \leq \frac{sun(G-S)+1}{2sun(G-S)} = \frac{1}{2} + \frac{1}{2sun(G-S)} \\ &\leq \frac{1}{2} + \frac{1}{6} = \frac{2}{3}, \end{aligned}$$

which contradicts $t(G) > \frac{2}{3}$. Theorem 4 is proved. ■

3. Isolated Toughness and $P_{\geq 3}$-Factor Covered Graphs

Yang, Ma and Liu [14] introduced a new parameter, isolated toughness of a graph G, denoted by $I(G)$, which is defined as

$$I(G) = \min\left\{\frac{|S|}{i(G-S)} : S \subseteq V(G), i(G-S) \geq 2\right\},$$

if G is not complete; otherwise, $I(G) = +\infty$. In the following, we investigate the relationship between isolated toughness and $P_{\geq 3}$-factor covered graphs, and obtain an isolated toughness condition for the existence of $P_{\geq 3}$-factor covered graphs. Our main result is the following theorem.

Theorem 6. *Let G be a connected graph with at least three vertices. If $I(G) > \frac{5}{3}$, then G is a $P_{\geq 3}$-factor covered graph.*

Remark 7. Let us show that $I(G) > \frac{5}{3}$ in Theorem 6 cannot be replaced by $I(G) \geq \frac{5}{3}$. We show this by constructing a graph $G = K_2 \vee (3K_2)$. It is easy to see that $I(G) = \frac{5}{3}$. Set $S = V(K_2)$, and so $|S| = 2$. Then by the definition of $\varepsilon(S)$, we obtain $\varepsilon(S) = 2$. Hence, we obtain

$$sun(G-S) = 3 > 2 = 2|S| - \varepsilon(S).$$

In terms of Theorem 2, G is not a $P_{\geq 3}$-factor covered graph.

***Proof of Theorem* 6.** If G is complete, obviously G is a $P_{\geq 3}$-factor covered graph as $|V(G)| \geq 3$. In the following, we assume that G is not complete. Suppose that G satisfies the hypothesis of Theorem 6, but it is not a $P_{\geq 3}$-factor covered graph. Then by Theorem 2, there exists a subset S of $V(G)$ satisfying

$$sun(G-S) \geq 2|S| - \varepsilon(S) + 1. \tag{6}$$

We shall consider three cases by the value of $|S|$ and derive a contradiction in each case.

Case 1. $|S| = 0$. According to the definition of $\varepsilon(S)$, we have $\varepsilon(S) = 0$. Combining this with (6), we obtain

$$sun(G) \geq 1. \tag{7}$$

Note that since $sun(G) \leq \omega(G)$ and G is connected, we have

$$sun(G) \leq \omega(G) = 1. \tag{8}$$

It follows from (7) and (8) that

$$sun(G) = \omega(G) = 1. \tag{9}$$

By (9), $|V(G)| \geq 3$ and the definition of sun, it is easy to see that G is a big sun. We use R to denote the factor-critical subgraph of G and set $U = V(R)$. Apparently, $i(G-U) = |U| \geq 3$. Then by $I(G) > \frac{5}{3}$ and the definition of $I(G)$, we have

$$\frac{5}{3} < I(G) \leq \frac{|U|}{i(G-U)} = 1,$$

which is a contradiction.

Case 2. $|S| = 1$. Clearly, $\varepsilon(S) \leq 1$. In terms of (6), we obtain

$$sun(G-S) \geq 2|S| - \varepsilon(S) + 1 \geq 2. \tag{10}$$

Assume that there exist a isolated vertices, b K_2's and c big sun components $H_1, H_2, \ldots, H_c$, where $|V(H_i)| \geq 6$, in $G-S$. Thus, it follows from (10) that

$$sun(G-S) = a + b + c \geq 2. \tag{11}$$

We choose one vertex from every K_2 component of $G-S$, and use X to denote the set of such vertices. For every H_i, we denote the factor-critical subgraph of H_i by R_i. We choose one vertex $y_i \in V(R_i)$ for $1 \leq i \leq c$, and write $Y = \{y_1, y_2, \ldots, y_c\}$. Apparently, we obtain

$$i(G - (S \cup X \cup Y)) = a + b + c \geq 2.$$

In terms of (6), (11), the definition of $I(G)$, $\varepsilon(S) \leq 1$ and $I(G) > \frac{5}{3}$, we have

$$\frac{5}{3} < I(G) \leq \frac{|S \cup X \cup Y|}{i(G-(S \cup X \cup Y))} = \frac{|S|+b+c}{a+b+c} = \frac{|S|+sun(G-S)-a}{sun(G-S)}$$

$$\leq \frac{|S|+sun(G-S)}{sun(G-S)} \leq \frac{\frac{sun(G-S)+\varepsilon(S)-1}{2}+sun(G-S)}{sun(G-S)} \leq \frac{3}{2},$$

which is a contradiction.

Case 3. $|S| \geq 2$. Note that $\varepsilon(S) \leq 2$. Combining this with (6), we obtain

$$sun(G-S) \geq 2|S| - \varepsilon(S) + 1 \geq 2|S| - 1 \geq 3. \tag{12}$$

Assume that there exist a isolated vertices, b K_2's and c big sun components $H_1, H_2, \dots, H_c$, where $|V(H_i)| \geq 6$, in $G-S$. Thus, we have $sun(G-S) = a+b+c$. We choose one vertex from each K_2 component of $G-S$, and denote the set of such vertices by X. We use R_i to denote the factor-critical subgraph of H_i for each H_i, and set $Y_i = V(R_i)$. Obviously, $|X| = b$ and $i(H_i - Y_i) = |Y_i| = \frac{|V(H_i)|}{2}$. Put $Y = \bigcup_{i=1}^{c} Y_i$. Then by (12) we obtain

$$\begin{aligned} i(G-(S \cup X \cup Y)) &= a+b+\sum_{i=1}^{c}|Y_i| = a+b+\sum_{i=1}^{c}\frac{|V(H_i)|}{2} \\ &\geq a+b+c = sun(G-S) \geq 3. \end{aligned}$$

Combining this with $I(G) > \frac{5}{3}$ and the definition of $I(G)$, we have

$$\frac{5}{3} < I(G) \leq \frac{|S \cup X \cup Y|}{i(G-(S \cup X \cup Y))} = \frac{|S|+b+\sum_{i=1}^{c}\frac{|V(H_i)|}{2}}{a+b+\sum_{i=1}^{c}\frac{|V(H_i)|}{2}}$$

that is,

$$3|S| > 5a + 2b + 2\sum_{i=1}^{c}\frac{|V(H_i)|}{2}. \tag{13}$$

Note that $|V(H_i)| \geq 6$ and $sun(G-S) = a+b+c$. According to (12) and (13), we have

$$\begin{aligned} 3|S| &> 5a+2b+2\sum_{i=1}^{c}\frac{|V(H_i)|}{2} \geq 5a+2b+6c \\ &\geq 2(a+b+c) = 2sun(G-S) \geq 2(2|S|-1), \end{aligned}$$

which implies

$$|S| < 2,$$

which contradicts $|S| \geq 2$. This completes the proof of Theorem 6. ■

4. Regular Graphs and $P_{\geq 3}$-Factor Covered Graphs

Kaneko [8] showed a condition for a regular graph to have a $P_{\geq 3}$-factor.

Theorem 8 (Kaneko [8])**.** *Every regular graph G with degree $r \geq 2$ admits a $P_{\geq 3}$-factor.*

In this section, we mainly study the relationship between regular graphs and $P_{\geq 3}$-factor covered graphs, and obtain a sufficient condition for a regular graph to be a $P_{\geq 3}$-factor covered graph. Our main result is shown in the following, and it is an improvement of Theorem 8.

Theorem 9. *Every regular graph G with degree $r \geq 2$ is a $P_{\geq 3}$-factor covered graph.*

Proof. Without loss of generality, we may assume that G is connected. Otherwise, we consider each connected component of G.

Suppose that G is not a $P_{\geq 3}$-factor covered graph. Then by Theorem 2, there exists a subset S of $V(G)$ satisfying

$$sun(G - S) \geq 2|S| - \varepsilon(S) + 1. \tag{14}$$

Claim 1. $S \neq \emptyset$.

Proof. If $S = \emptyset$, then $\varepsilon(S) = 0$. By (14), we have

$$sun(G) \geq 1.$$

On the other hand, G is connected, and so $sun(G) \leq \omega(G) \leq 1$. Thus, we obtain

$$sun(G) = 1.$$

Obviously, G itself is a sun. Note that $r \geq 2$. Hence, $G \neq K_1$ and $G \neq K_2$. Thus, G is a big sun, which contradicts that G is a regular graph with degree $r \geq 2$. This completes the proof of Claim 1. □

Claim 2. $sun(G - S) \geq 2$.

Proof. According to Claim 1, we have $|S| \geq 1$.

If $|S| = 1$, then $\varepsilon(S) \leq 1$. It follows from (14) that

$$sun(G - S) \geq 2|S| - \varepsilon(S) + 1 \geq 2|S| = 2.$$

In the following, we consider $|S| \geq 2$. In this case, $\varepsilon(S) \leq 2$. Then by (14), we obtain

$$sun(G - S) \geq 2|S| - \varepsilon(S) + 1 \geq 2|S| - 1 \geq 3 > 2.$$

This completes the proof of Claim 2. □

In the following, we assume that there exist a isolated vertices, b K_2's and c big sun components $H_1, H_2, \ldots, H_c$, where $|V(H_i)| \geq 6$, in $G - S$. In terms of Claim 2, we have

$$sun(G - S) = a + b + c \geq 2. \tag{15}$$

For any $x \in V(bK_2)$, the degree of x in bK_2 is 1. For each H_i, H_i has at least three vertices of degree exactly one. Note that G is a regular graph with degree $r \geq 2$. Thus, we obtain

$$ar + 2b(r-1) + 3c(r-1) \leq r|S|.$$

Combining this with (14), (15) and $\varepsilon(S) \leq 2$, we have

$$\begin{aligned} ar + 2b(r-1) + 3c(r-1) &\leq r|S| \leq \frac{r}{2}\left(sun(G-S) + \varepsilon(S) - 1\right) \\ &\leq \frac{r}{2}\left(sun(G-S) + 1\right) = \frac{r}{2}(a + b + c + 1), \end{aligned}$$

that is,

$$ar + 3br + 5cr - r \leq 4b + 6c. \tag{16}$$

It follows from (15), (16) and $r \geq 2$ that

$$\begin{aligned} 2a + 6b + 10c - 2 &= 2(a + 3b + 5c - 1) \leq r(a + 3b + 5c - 1) \\ &= ar + 3br + 5cr - r \leq 4b + 6c, \end{aligned}$$

which implies

$$a + b + 2c \leq 1.$$

Note that $c \geq 0$. Hence, we obtain

$$a + b + c \leq 1,$$

which contradicts (15). The proof of Theorem 9 is complete. ■

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