

Discussiones Mathematicae
Graph Theory 37 (2017) 745–754
doi:10.7151/dmgt.1961

A NOTE ON THE LOCATING-TOTAL DOMINATION IN GRAPHS

MIRKA MILLER (POSTHUMOUS)

School of Mathematical and Physical Sciences
University of Newcastle, Australia
Department of Mathematics
University of West Bohemia, Pilsen, Czech Republic

e-mail: mirka.miller@gmail.com

R. SUNDARA RAJAN

Department of Mathematics, Anna University, Chennai-600 025, India

e-mail: vprsundar@gmail.com

R. JAYAGOPAL, INDRA RAJASINGH

School of Advanced Sciences, VIT University, Chennai-600 127, India

e-mail: jgopal89@gmail.com
indrarajasinh@yahoo.com

AND

PAUL MANUEL

Department of Information Science, Kuwait University, Safat, Kuwait

e-mail: pauldmanuel@gmail.com

Abstract

In this paper we obtain a sharp (improved) lower bound on the locating-total domination number of a graph, and show that the decision problem for the locating-total domination is NP-complete.

Keywords: dominating set, total dominating set, locating-dominating set, locating-total dominating set, regular graphs.

2010 Mathematics Subject Classification: 05C69.

1. INTRODUCTION

A set S of vertices in a graph G is called a *dominating set* of G if every vertex in $V(G) \setminus S$ is adjacent to some vertex in S . The set S is said to be a *total dominating set* of G if every vertex in $V(G)$ is adjacent to some vertex in S . The domination problem is to determine the minimum cardinality of all *dominating sets* in G . Similarly, the total domination problem is the problem of determining the minimum cardinality of such sets in G . A *locating-dominating set* in a connected graph G is a dominating set S of G such that for every pair of vertices u and v in $V(G) \setminus S$, $N(u) \cap S \neq N(v) \cap S$. The minimum cardinality of a locating-dominating set of G is the *locating-domination number* $\gamma^L(G)$ [6]. A *locating-total dominating set* in a connected graph G is a total dominating set S of G such that for every pair of vertices u and v in $V(G) \setminus S$, $N(u) \cap S \neq N(v) \cap S$. The minimum cardinality of a locating total-dominating set of G is the *locating-total domination number* $\gamma_t^L(G)$ [6]. Determining if an arbitrary graph has a dominating set and locating-dominating set of a given size are well-known *NP*-complete problems [1, 5].

Total domination plays a role in the problem of placing monitoring devices in a system in such a way that every site in the system, including the monitors, is adjacent to a monitor site so that, if a monitor goes down, then an adjacent monitor can still protect the system. Installing the minimum number of expensive sensors in the system which will transmit a signal at the detection of faults and uniquely determine the location of the faults motivates the concept of locating-dominating sets and locating-total dominating sets [6].

The locating-total domination problem has been discussed for trees [2, 3], cubic graphs and grid graphs [8], corona and composition of graphs [10], claw-free cubic graphs [7], and so on.

The paper is organized as follows. In Section 2, we obtain an improved bound for locating-total domination of regular graphs. Further we prove that the bound is tight for certain families of regular graphs. In Section 3, we prove that the locating-total domination problem is *NP*-complete.

2. LOWER BOUND FOR THE LOCATING-TOTAL DOMINATION NUMBER

All graphs considered in this paper are simple and connected.

Let $G = (V, E)$ be a graph and $S \subseteq V(G)$, a dominating set of G . By the *shadow* of a vertex $u \in V(G)$ on S , we mean the set $S_u = S \cap N[u]$ where $N[u] = N(u) \cup \{u\}$. The *profile* of $u \in V(G)$ is defined to be the $(d_G(u) + 1)$ -tuple $\pi(u)$ with entries $|S_x|$ where $x \in N[u]$, in ascending order. The *share* of a vertex

$u \in S$ in S is defined by

$$\gamma(u, S) = \sum_{x \in N[u]} \frac{1}{|S_x|}.$$

When the set S is clear from the context, we refer to $\gamma(u, S)$ simply as the share of u and denote it by $\gamma(u)$.

The following lemma is a powerful tool in obtaining lower bounds on various flavors of domination numbers. This result was given in [11].

Lemma 2.1 [11]. *Let G be a graph of order n and let S be a dominating set of G . Then $\sum_{u \in S} \gamma(u) = n$.*

In what follows, we give an improved lower bound for $\gamma_t^L(G)$ when G is regular.

2.1. Improved lower bound for regular graphs

Henning *et al.* [8] have proved that the locating-total domination number for a graph G satisfies the inequalities $\gamma_t^L(G) \geq \lfloor \log_2 n \rfloor$ and $\gamma_t^L(G) \geq (\text{diameter}(G) + 1)/2$.

In this section, we have obtained an improved lower bound for the locating-total domination number for regular graphs. For proving the main result, we need the following.

Lemma 2.2. *Let S be a locating-total dominating set of a k -regular graph G of order n , for some positive integer $k \geq 2$. Then $\gamma(u) \leq \frac{k+2}{2}$, for each $u \in S$.*

Proof. Let $u \in S$. Since S is a total dominating set, at least one vertex v in $N(u)$ belongs to S . Now for any two distinct vertices x and y of $N[u]$ we claim that $|S_x| = |S_y| = 1$ is not possible. For, if $|S_x| = |S_y| = 1$, then $N(x) \cap S = N(y) \cap S$, a contradiction. Therefore $|S_x| = 1$ for at most one vertex x of $N[u]$. For all vertices $y \neq x$ in $N[u]$, $|S_y| \geq 2$. Hence for all vertices $y \neq x$ in $N[u]$, $\frac{1}{|S_y|} \leq \frac{1}{2}$. Thus we have $\gamma(u) = \sum_{w \in N[u]} \frac{1}{|S_w|} \leq 1 + k(\frac{1}{2}) = \frac{k+2}{2}$. ■

Theorem 2.3. *Let G be a k -regular graph of order n . Then $\gamma_t^L(G) \geq \left\lceil \frac{2n}{k+2} \right\rceil$.*

Proof. Let S be a locating-total dominating set of G . By Lemma 2.2, we have $\gamma(u) \leq \frac{k+2}{2}$, for all $u \in S$. By Lemma 2.1, $n = \sum_{u \in S} \gamma(u) \leq \frac{k+2}{2} |S|$. Therefore $|S| \geq \left\lceil \frac{2n}{k+2} \right\rceil$. ■

Remark 1. For a given k , there exists an integer n , n large, such that $\left\lceil \frac{2n}{k+2} \right\rceil > \lfloor \log_2 n \rfloor$. Such a pair of numbers is denoted by $n(k)$. Thus our bound obtained in Theorem 2.3 is better than the bound obtained by Henning *et al.* [8].

In the sequel we prove that the lower bound obtained in Theorem 2.3 is sharp for extended cycle-of-ladders and circulant networks. Without loss of generality we refer to the vertices in these graphs by their labels.

2.2. γ_t^L of extended cycle-of-ladder $ECL(2l, s)$

In [4], Fang introduced a network called cycle-of-ladder and proved that it is a spanning subgraph of the hypercube network, thereby proving that hypercube network is bipancyclic. In this section, we derive a new network from cycle-of-ladder and call it the extended cycle-of-ladder network.

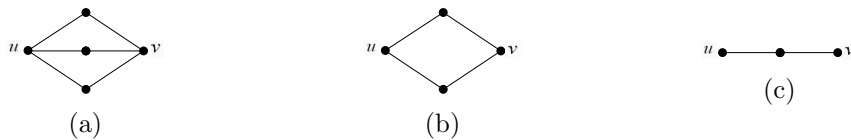


Figure 1. Illustrates the proof of Proposition 2.

Definition [9]. The n -ladder graph L of length n is defined as $P_2 \times P_{n+1}$, where P_{n+1} is a path on $n + 1$ vertices, $n \geq 1$.

The graph obtained via this definition has the advantage of looking like a ladder having two rails and $n + 1$ rungs between them. The length of the ladder is defined as n .

Definition [4]. A cycle-of-ladder is a graph containing a cycle C_b of length $2l$ called the bone cycle and l ladders L_1, L_2, \dots, L_l with $R_b(1), R_b(2), \dots, R_b(l)$ as the bottom rungs such that $R_b(i)$'s are respectively the alternate edges in C_b , $1 \leq i \leq l$. We denote the cycle-of-ladder as $CL(2l, s)$, where l and s represent the number of ladders and the length of each ladder, respectively.

For convenience we label the vertices of L_i as $l_{j,1}^i$ and $l_{j,2}^i$ where $0 \leq j \leq s$ and $1 \leq i \leq l$ in $CL(2l, s)$. Figure 2(a) illustrates $(l_{0,1}^1, l_{0,2}^1, l_{0,1}^2, l_{0,2}^2, l_{0,1}^3, l_{0,2}^3, l_{0,1}^4, l_{0,2}^4, l_{0,1}^1)$ as the bone cycle and the edges $(l_{0,1}^1, l_{0,2}^1), (l_{0,1}^2, l_{0,2}^2), (l_{0,1}^3, l_{0,2}^3), (l_{0,1}^4, l_{0,2}^4)$ as $R_b(1), R_b(2), R_b(3)$ and $R_b(4)$, respectively.

We add l number of edges to $CL(2l, s)$ to obtain a 3-regular graph and call it the extended cycle-of-ladder $ECL(2l, s)$.

Definition. The extended cycle-of-ladder $ECL(2l, s)$ is obtained from $CL(2l, s)$ by adding edges between $l_{s,2}^i$ and $l_{s,1}^{i+1}$, where $1 \leq i \leq l - 1$, and between $l_{s,2}^l$ and $l_{s,1}^1$.

Proposition 2. Let G be an extended cycle-of-ladder $ECL(2l, s)$ with $l \equiv 0 \pmod{2}$ and $s \equiv 4 \pmod{5}$. Then $\gamma_t^L(ECL(2l, s)) = 4l(s + 1)/5$.

Proof. Label the vertices of $L(i)$ as $l_{j,1}^i$ and $l_{j,2}^i$ where $0 \leq j \leq s$ and $1 \leq i \leq l$ in $ECL(2l, s)$. See Figure 2(b). Since $s \equiv 4 \pmod{5}$, $s + 1$ is a multiple of 5. We have $s + 1$ rungs in each ladder L_i , $1 \leq i \leq l$. Partition the $s + 1$ rungs into sets $P_1, P_2, \dots, P_{(s+1)/5}$ of five consecutive rungs beginning from the bottom rung in each ladder. Let S contain the vertices in the second and fourth rungs of each partition. In other words, $S = \bigcup_{1 \leq i \leq \lceil s/5 \rceil} \bigcup_{1 \leq j \leq l/2} \left\{ l_{5i-4,1}^{2j-1}, l_{5i-2,1}^{2j-1}, l_{5i-4,2}^{2j-1}, l_{5i-2,2}^{2j-1}, l_{5i-4,1}^{2j}, l_{5i-2,1}^{2j}, l_{5i-4,2}^{2j}, l_{5i-2,2}^{2j} \right\}$. We claim that S is a minimum locating-total dominating set of $ECL(2l, s)$. Clearly S is a total dominating set. We have only to prove that S is a locating-total dominating set of $ECL(2l, s)$. Let $u, v \in V \setminus S$. If u and v are in different ladders, then $N(u) \cap S \neq N(v) \cap S$. Suppose u and v are in the same ladder, say L . Suppose $N(u) \cap S = N(v) \cap S$. If $|N(u) \cap S| = |N(v) \cap S| = 3$, then u, v and the three vertices adjacent to both u and v induce a subgraph shown in Figure 1(a), which is not possible by the definition of extended cycle-of-ladder. If $|N(u) \cap S| = |N(v) \cap S| = 2$, then u, v and the two vertices adjacent to both u and v induce a subgraph shown in Figure 1(b), which is not possible by the choice of S . Now $|N(u) \cap S| = |N(v) \cap S| = 1$ is not possible (see Figure 1(c)), since at least one of u, v has two vertices of S adjacent to it, contradicting $N(u) \cap S = N(v) \cap S$. Thus S is a locating-total dominating set in $ECL(2l, s)$. Now $|S| = 8(\lceil s/5 \rceil)(l/2) = 4l(s + 1)/5$. By Theorem 2.3, $\gamma_t^L(ECL(2l, s)) = 4l(s + 1)/5$. ■

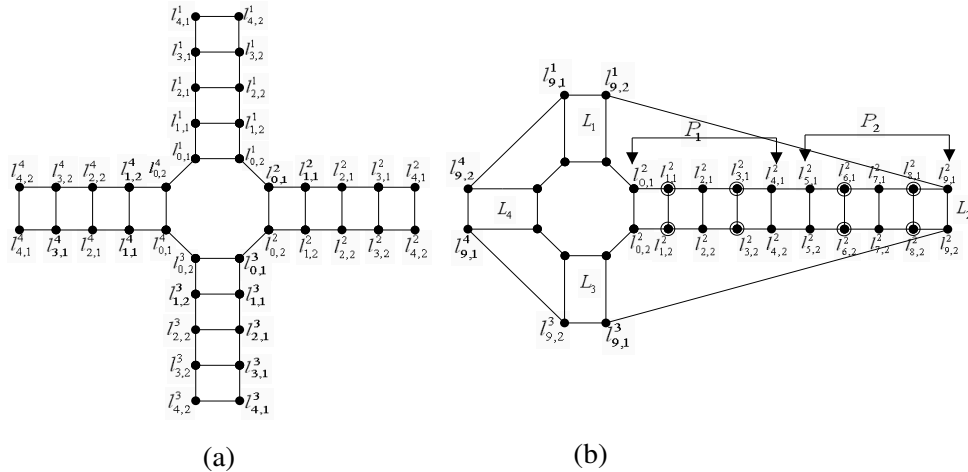


Figure 2. (a) $CL(8, 4)$.
 (b) Vertices in a locating-total dominating set of $ECL(8, 9)$ are circled.

2.3. γ_t^L of circulant graph $G(n, \pm\{1, 2\})$

Definition [12]. The undirected circulant graph $G(n, \pm S)$, where $S \subseteq \{1, 2, \dots, j\}$, $1 \leq j \leq \lfloor n/2 \rfloor$, the vertex set $V = \{0, 1, \dots, n - 1\}$ and the edge set $E = \{(i, k) : |k - i| \equiv s \pmod n, s \in S\}$.

For brevity, we use the label $0, 1, 2, \dots, n - 1$ as $1, 2, \dots, n$ in $G(n, \pm S)$.

Proposition 3. *Let G be a circulant graph $G(n, \pm\{1, 2\})$ where $n \geq 7$. Then $\gamma_t^L(G(n, \pm\{1, 2\})) = \lceil n/3 \rceil$ if $n \equiv 0, 1, 2, 4 \pmod 6$.*

Proof. Label the vertices of $G(n, \pm\{1, 2\})$ from 1 to n , sequentially with clockwise sense. We begin with the case when $n \equiv 0 \pmod 6$, where all labels are taken modulo n . Let $S = \bigcup_{1 \leq k \leq n/6} \{n - 6k + 3, n - 6k + 1\}$, $1 \leq k \leq n/6$. We claim that S is a locating-total dominating set of $G(n, \pm\{1, 2\})$. Let $N_{V \setminus S}(S)$ denote the set of all neighborhood in $V \setminus S$ of members of S . For $1 \leq k \leq n/6$, it is easy to see that $N_{V \setminus S}(S) = N(S) \cap V \setminus S = V \setminus S$.

Moreover $(n - 6k + 3, n - 6k + 1)$ is an edge in $G(n, \pm\{1, 2\})$. Therefore S is a total dominating set in $G(n, \pm\{1, 2\})$. We have only to show that S is a locating-total dominating set. For $1 \leq k \leq n/6$, $N(n - 6k + 2) \cap S = \{n - 6k + 3, n - 6k + 1\}$, $N(n - 6k + 4) \cap S = \{n - 6k + 3\}$, $N(n - 6k + 5) \cap S = \{n - 6k + 3, n - 6k + 7\}$ and $N(n - 6k + 6) \cap S = \{n - 6k + 7\}$, which are all distinct. Now $|S| = 2(n/6) = \lceil n/3 \rceil$. See Figure 3(a). By Theorem 2.3, $\gamma_t^L(G(n, \pm\{1, 2\})) = 2n/(k+2) = 2n/(4+2) = n/3$, when $n \equiv 0 \pmod 6$.

When $n \equiv 1, 2 \pmod 6$, $S = \bigcup_{1 \leq k \leq n/6} \{n - 6k + 3, n - 6k + 1\} \cup \{1\}$, $1 \leq k \leq \lfloor n/6 \rfloor$; and when $n \equiv 4 \pmod 6$, $S = \bigcup_{1 \leq k \leq n/6} \{n - 6k + 3, n - 6k + 1\} \cup \{1, 3\}$, $1 \leq k \leq \lfloor n/6 \rfloor$ are respectively the minimum locating-total dominating sets in $G(n, \pm\{1, 2\})$. Thus by Theorem 2.3, $\gamma_t^L(G(n, \pm\{1, 2\})) = \lceil n/2 \rceil$, when $n \equiv 0, 1, 2, 4 \pmod 6$. See Figure 3(b). ■

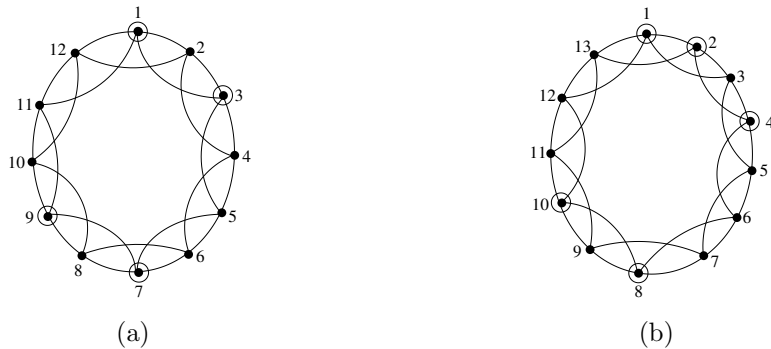


Figure 3. (a) Vertices in a locating-total dominating set of $G(12, \pm\{1, 2\})$ and (b) vertices in a locating-total dominating set of $G(13, \pm\{1, 2\})$ are circled.

3. LOCATING-TOTAL DOMINATION PROBLEM IS NP-COMPLETE

The locating-domination problem and locating-total domination problem are not equivalent. In other words, it is not possible to derive a minimum locating-dominating set from a minimum locating-total dominating set and vice-versa. For example, consider the graph G shown in Figure 4. In G the minimum locating-dominating set $T = \{2, 5, 7\}$ and hence $\gamma^L(G) = 3$ (see Figure 4(a)). Now, in G the minimum locating-total dominating set $S = \{2, 3, 7, 8\}$ and hence $\gamma_t^L(G) = 4$ (see Figure 4(b)). Locating-domination problem is NP-complete [1]. In this section we prove locating-total domination problem is NP-complete.

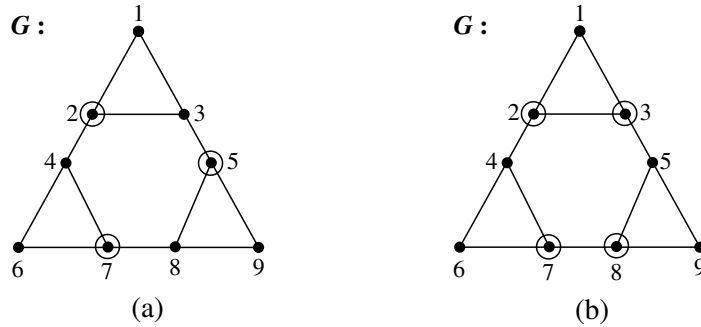


Figure 4. (a) Vertices in a locating-dominating set of G are circled.
 (b) Vertices in a locating-total dominating set of G are circled.

Theorem 3.1. *The following decision problem is NP-complete:*

Name: locating-total dominating set (LTDS).

Instance: a connected graph $G = (V, E)$ and an integer $k \leq |V|$.

Question: is there a locating-total dominating set $S \subseteq V$ of size at most k ?

Proof. We polynomially reduce 3-SAT to LTDS. We consider any instance of 3-SAT, $\mathbb{C} = \{C_1, C_2, \dots, C_m\}$ over the set of variables $X = \{x_1, x_2, \dots, x_n\}$. For each variable x_i of X , we construct the graph $G_{x_i} = (V_{x_i}, E_{x_i})$ with $V_{x_i} = \{a_i, b_i, c_i, d_i, e_i, x_i, \bar{x}_i\}$ and $E_{x_i} = \{a_i x_i, a_i \bar{x}_i, a_i c_i, b_i c_i, c_i x_i, c_i \bar{x}_i, d_i x_i, d_i \bar{x}_i, d_i e_i\}$, $1 \leq i \leq n$.

Next for each clause $C_j = \{u_{j,1}, u_{j,2}, u_{j,3}\}$, we construct the graph $G_{C_j} = (V_{C_j}, E_{C_j})$, with $V_{C_j} = \{\alpha_j, \beta_j, \gamma_j, \mu_j\}$ and $E_{C_j} = \{\alpha_j \beta_j, \beta_j \gamma_j, \gamma_j \mu_j, \mu_j \eta_j\}$, $1 \leq j \leq m$.

Finally, given formula $F = C_1 \wedge C_2 \wedge \dots \wedge C_m$ we construct $G = (V, E)$ with

$$V = \left(\bigcup_{i=1}^n V_{x_i} \right) \cup \left(\bigcup_{i=1}^m V_{C_j} \right),$$

$$E = \left(\bigcup_{i=1}^n E_{x_i} \right) \cup \left(\bigcup_{i=1}^m E_{C_j} \right) \cup \left(\bigcup_{i=1}^n \{\alpha_j u_{j,1}, \alpha_j u_{j,2}, \alpha_j u_{j,3}\} \right).$$

We set $k = 3n + 2m$; we see that $|V| = 7n + 5m$ and $|E| = 9n + 8m$. See Figure 5 with $n = 3$ and $m = 2$. In Figure 5, $F = C_1 \wedge C_2$, where $C_1 = (u_{1,1} \vee u_{1,2} \vee u_{1,3}) = (x_1 \vee x_2 \vee x_3)$ and $C_2 = (u_{2,1} \vee u_{2,2} \vee u_{2,3}) = (x_1 \vee \bar{x}_2 \vee \bar{x}_3)$.

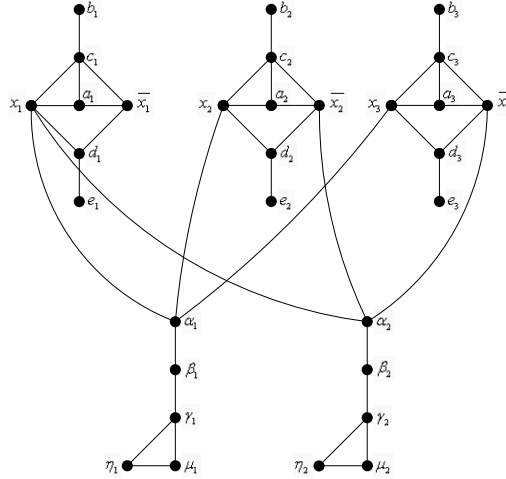


Figure 5. Graph of formula $F = (x_1 \vee x_2 \vee x_3) \wedge (x_1 \vee \bar{x}_2 \vee \bar{x}_3)$.

(i) If F is satisfied, we can construct a locating-total dominating set $S \subseteq V$, of size k , as follows. For all j and i where $1 \leq j \leq m, 1 \leq i \leq n$, let S contain $\gamma_j, \mu_j, c_i, d_i$, and whichever of x_i and \bar{x}_i that has been set True. The set S thus constructed has size $3n + 2m = k$. Clearly S is a total dominating set of G . We have only to show that S is a locating-total dominating set. Without loss of generality, assume that $x_i \in S$; then $N(a_i) \cap S = \{c_i, x_i\}$, $N(b_i) \cap S = \{c_i\}$, $N(\bar{x}_i) \cap S = \{c_i, d_i\}$, $N(\bar{e}_i) \cap S = \{d_i\}$; moreover, $N(\beta_j) \cap S = \{\gamma_j\}$, $N(\eta_j) \cap S = \{\gamma_j, \mu_j\}$ and using the assumption that each clause contains at least one true literal, at least one vertex of type x_i or \bar{x}_i will be in $N(\alpha_j) \cap S$.

(ii) Now we assume that there is a subset S of V , of size at most k , which is a locating-total dominating set. It is clear that for all j , either $\{\gamma_j, \mu_j\} \in S$ or $\{\gamma_j, \eta_j\} \in S$. Suppose not. If $\{\beta_j, \gamma_j\} \in S$, then $N(\mu_j) \cap S = \{\gamma_j\} = N(\eta_j) \cap S$ and, if $\{\alpha_j, \beta_j\} \in S$, then μ_j and η_j are not dominated and, if $\{\mu_j, \eta_j\} \in S$, then β_j is not dominated. Thus in all cases, either $\{\gamma_j, \mu_j\} \in S$ or $\{\gamma_j, \eta_j\} \in S$ and α_j must be dominated by another vertex.

Let us now consider the sets $S \cap V_{x_i}$; we claim that at least three elements in V_{x_i} are necessary to make $N(u) \cap S \neq N(v) \cap S$ for all u and v in $V_{x_i} \setminus S$, and that, moreover, if we manage with exactly three, then exactly one of x_i belongs to S . Indeed, suppose first that x_i or \bar{x}_i are in S . Then since two more elements are necessary in V_{x_i} to locate b_i and e_i , either $|S \cap V_{x_i}| \geq 4$ or $|S \cap V_{x_i}| = 3$ and exactly one of x_i and \bar{x}_i belongs to S . Suppose next that neither x_i nor \bar{x}_i are in S . Then, in order to locate and separate a_i, b_i and c_i , and d_i and e_i , at

least three elements in $V_{x_i} \setminus \{x_i, \bar{x}_i\}$ are necessary. Now if $\{a_i, c_i, d_i\} \subset S$, then $N(x_i) \cap S = N(\bar{x}_i) \cap S = \{a_i, c_i, d_i\}$; this implies that x_i or \bar{x}_i is located by a vertex of type α . This however contradicts the assumption on the size of $|S|$, since already $3n + 2m$ other vertices necessarily belong to S .

Now, we know that S contains exactly k elements; in particular, exactly two vertices belong to V_{C_j} and exactly three vertices are in V_{x_i} , with exactly one of x_i and \bar{x}_i in S .

Thus, setting $x_i = \text{True}$ if $S \cap \{x_i, \bar{x}_i\} = \{x_i\}$ and $x_i = \text{False}$ if $S \cap \{x_i, \bar{x}_i\} = \{\bar{x}_i\}$ is a valid truth assignment for the variables of X . Now in order to locate α_j at least one vertex of type x_i or \bar{x}_i must be in S , corresponding to one of the three literals in the clause C_j . This means that C_j contains at least one true literal and it holds for all j . Hence we have a truth assignment which satisfies F . ■

We end the paper with the followings problems.

Problem 1. Can Theorem 3.1 be improved for bipartite graphs and chordal graphs?

Problem 2. Can improved bounds for locating-total domination number be obtained for interval graphs and split graphs?

Acknowledgements

The work of R. Sundara Rajan was supported by Dr D.S. Kothari Fellowship, No. F.4-2/2006 (BSR)/MA/15-16/0011, University Grants Commission, Government of India. The work of Indra Rajasingh was supported by Project No. SR/S4/MS: 846/13, Department of Science and Technology, SERB, Government of India.

The authors would like to thank the anonymous referees for their comments and suggestions. These comments and suggestions were very helpful for improving the quality of this paper.

REFERENCES

- [1] I. Charon, O. Hudry and A. Lobstein, *Minimizing the size of an identifying or locating-dominating code in a graph is NP-hard*, Theoret. Comput. Sci. **290** (2003) 2109–2120.
doi:10.1016/S0304-3975(02)00536-4
- [2] M. Chellali, *On Locating and differentiating-total domination in trees*, Discuss. Math. Graph Theory **28** (2008) 383–392.
doi:10.7151/dmgt.1414
- [3] X. Chen and M.Y. Sohn, *Bounds on the locating-total domination number of a tree*, Discrete Appl. Math. **159** (2011) 769–773.
doi:10.1016/j.dam.2010.12.025

- [4] J.-F. Fang, *The bipancycle-connectivity of the hypercube*, Inform. Sci. **178** (2008) 4679–4687.
- [5] M.R. Garey and D.S. Johnson, *Computers and Intractability: A Guide to the Theory of NP-Completeness* (W.H. Freeman & Company Publisher, San Francisco, US, 1979).
- [6] T.W. Haynes, M.A. Henning and J. Howard, *Locating and total dominating sets in trees*, Discrete Appl. Math. **154** (2006) 1293–1300.
doi:10.1016/j.dam.2006.01.002
- [7] M.A. Henning and C. Löwenstein, *Locating-total domination in claw-free cubic graphs*, Discrete Math. **312** (2012) 3107–3116.
doi:10.1016/j.disc.2012.06.024
- [8] M.A. Henning and N.J. Rad, *Locating-total domination in graphs*, Discrete Appl. Math. **160** (2012) 1986–1993.
doi:10.1016/j.dam.2012.04.004
- [9] H. Hosoya and F. Harary, *On the matching properties of three fence graphs*, J. Math. Chem. **12** (1993) 211–218.
doi:10.1007/BF01164636
- [10] B.N. Omamalin, *Locating total dominating sets in the join, corona and composition of graphs*, Appl. Math. Sci. **8** (2014) 2363–2374.
doi:10.12988/ams.2014.43205
- [11] P.J. Slater, *Fault-tolerant locating-dominating sets*, Discrete Math. **249** (2002) 179–189.
doi:10.1016/S0012-365X(01)00244-8
- [12] J. Xu, *Topological Structure and Analysis of Interconnection Networks* (Kluwer Academic Publishers, London, UK, 2001).
doi:10.1007/978-1-4757-3387-7

Received 6 January 2016

Revised 20 June 2016

Accepted 20 June 2016