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UNIQUE-MAXIMUM COLORING OF PLANE GRAPHS

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Abstract

A unique-maximum k-coloring with respect to faces of a plane graph G is a coloring with colors $1, \ldots, k$ so that, for each face α of G, the maximum color occurs exactly once on the vertices of α. We prove that any plane graph is unique-maximum 3-colorable and has a proper unique-maximum coloring with 6 colors.

Keywords: plane graph, weak-parity coloring, unique-maximum coloring.

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