Discussiones Mathematicae
Graph Theory 35 (2015) 95–104
doi:10.7151/dmgt.1782

GRAPHIC SPLITTING OF COGRAPHIC MATROIDS

Naiyer Pirouz

Department of Mathematics, University of Pune
Pune - 411007, India

e-mail: naiyer.pirouz@gmail.com

Abstract

In this paper, we obtain a forbidden minor characterization of a cographic matroid M for which the splitting matroid $M_{x,y}$ is graphic for every pair x, y of elements of M.

Keywords: binary matroid, graphic matroid, cographic matroid, minor, splitting operation.

2010 Mathematics Subject Classification: 05B35, 05C50, 05C83.

1. Introduction

Fleischner [3] introduced the idea of splitting a vertex of degree at least three in a connected graph and used the operation to characterize Eulerian graphs. Figure 1 shows the graph $G_{x,y}$ that is obtained from G by splitting away the edges x and y from the vertex v.

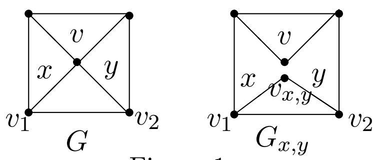


Figure 1

Welsh [11] proved that a binary matroid is Eulerian if and only if its dual is bipartite.

It is easy to see that a binary matroid M is Eulerian if and only if the sum of columns of A is zero, where A is a matrix over $GF(2)$ that represents M. Raghunathan *et al.* [7] proved that a binary matroid M is Eulerian if and only if $M_{x,y}$ is Eulerian for every pair of elements x and y.

The matroid notations and terminology used here will follow Oxley [6]. We adopt the convention that every graph mentioned in this paper is loopless and coloopless.

Raghunathan *et al.* [7] extended the splitting operation from graphs to binary matroids as follows:

Definition 1.1. Let $M = M[A]$ be a binary matroid and suppose $x, y \in E(M)$. Let $A_{x,y}$ be the matrix obtained from A by adjoining the row that is zero everywhere except for the entries of 1 in the columns labelled by x and y. The splitting matroid $M_{x,y}$ is defined to be the vector matroid of the matrix $A_{x,y}$.

Example 1.2. Consider the Fano matroid $F_7 = M$ on the set $E = \{1, 2, 3, 4, 5, 6, 7\}$. Let A denote the standard matrix representation with respect to the basis $B = \{1, 2, 3\}$ of M over $GF(2)$, so that

$$A = \begin{array}{c} \begin{matrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \end{matrix} \\ \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 \end{bmatrix} \end{array}.$$

Then splitting of M by the pair 2 and 4, i.e. the matroid $M_{2,4}$, is represented by the matrix

$$A_{2,4} = \begin{array}{c} \begin{matrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \end{matrix} \\ \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 \end{bmatrix} \end{array}.$$

Let $M(G)$ and $M^*(G)$ denote the cycle matroid and the cocycle matroid, respectively of a graph G. Various properties of a splitting matroid are obtained in [1, 2, 5, 7, 8, 9] and [10].

The splitting operation on a graphic matroid may not yield a graphic matroid. Shikare and Waphare [10] characterized graphic matroids whose splitting matroids for every pair of elements are also graphic. Also, cographicness of a matroid may not be preserved under the splitting operation. Borse, Shikare, and Dalvi [2] obtained a forbidden-minor characterization for this class.

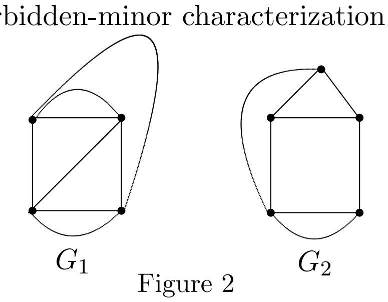


Figure 2

Further, the splitting operation on a cographic matroid may not yield a graphic matroid. In this paper, we characterize those cographic matroids M for which $M_{x,y}$ is graphic for every pair $x, y \in E(M)$. The following is the main theorem.

Theorem 1.3. *The splitting operation, by any pair of elements, on a cographic matroid yields a graphic matroid if and only if it has no minor isomorphic to any of the cycle matroids $M(G_1)$ and $M(G_2)$, where G_1 and G_2 are the graphs depicted in Figure 2.*

2. Graphic Splitting of Cographic Matroids

Firstly, we give some results which are used in the proof of the main result.

Lemma 2.1 [7]**.** *Let $M = (S, \mathcal{C})$ be a binary matroid on a set S together with the set $\mathcal{C}$ of circuits. Then $M_{x,y} = (S, \mathcal{C}')$ with $\mathcal{C}' = \mathcal{C}_0 \cup \mathcal{C}_1$, where $\mathcal{C}_0 = \{C \in \mathcal{C} : x, y \in C$ or $x \notin C, y \notin C\}$; and $\mathcal{C}_1 = \{C_1 \cup C_2 : C_1, C_2 \in \mathcal{C}, x \in C_1, y \in C_2, C_1 \cap C_2 = \emptyset$ and $C_1 \cup C_2$ contains no member of $\mathcal{C}_0\}$.*

Lemma 2.2 [5, 10]**.** *Let x and y be elements of a binary matroid M and let $r(M)$ denote the rank of M. Then the following statements hold.*

(i) *$M_{x,y} = M$ if and only if x and y are in series in M or both x and y are coloops in M,*

(ii) *$r(M_{x,y}) = r(M) + 1$ if and only if $M \neq M_{x,y}$,*

(iii) *if x_1, x_2 are in series in M, then they are in series in $M_{x,y}$.*

(iv) *If C^* is a cocircuit of M containing x, y with $|C^*| \geq 3$, then $C^* - \{x, y\}$ is a cocircuit of $M_{x,y}$; and*

(v) *$M_{x,y}/\{x\}$ is Eulerian if and only if M is Eulerian.*

Theorem 2.3 [6]**.** *A binary matroid is graphic if and only if it has no minor isomorphic to F_7, F_7^*, $M^*(K_5)$ or $M^*(K_{3,3})$.*

Theorem 2.4 [6]**.** *A binary matroid is cographic if and only if it has no minor isomorphic to F_7, F_7^*, $M(K_5)$ or $M(K_{3,3})$.*

Notation. For the sake of convenience, let $\mathcal{F} = \{F_7,\ F_7^*,\ M^*(K_5),\ M^*(K_{3,3})\}$.

Lemma 2.5. *Let M be a cographic matroid and let $x, y \in E(M)$ such that $M_{x,y}$ is not graphic. Then there is a minor N of M with $\{x, y\} \subset E(N)$ such that $N_{x,y}/\{x\} \cong F$ or $N_{x,y}/\{x, y\} \cong F$ for some $F \in \mathcal{F}$ and further, N has no non-trivial series class except possibly a series class which contains x and y.*

Proof. As in the proof of Theorem 2.3 in [10], there exists a minor N of M such that $N_{x,y}/\{x\} \cong F$ or $N_{x,y}/\{x, y\} \cong F$ for some $F \in \mathcal{F}$. If x and y are not in series in N, then N has no non-trivial series class. Suppose x and y are in series in N. Then, $N = N_{x,y}$. Since F does not have any 2-cocircuit, every 2-cocircuit of N must contain x or y. Hence N has at most one non-trivial series class. ■

Definition 2.6. Let M be a cographic matroid and let $F \in \mathcal{F}$. We say that M is minimal with respect to F if there exist two elements x and y of M such that $M_{x,y}/\{x\} \cong F$ or $M_{x,y}/\{x,y\} \cong F$ and further, M has no non-trivial series class except possibly a series class which contains x and y.

Corollary 2.7. *Let M be a cographic matroid. For any $x, y \in E(M)$, the matroid $M_{x,y}$ is graphic if and only if M has no minor isomorphic to a minimal matroid with respect to any $F \in \mathcal{F}$.*

Proof. The proof follows from Lemma 2.2 and Lemma 2.5. ■

Lemma 2.8. *Let M be a minimal matroid with respect to F for some $F \in \mathcal{F}$ and let x, y be two elements of M such that either $M_{x,y}/\{x\} \cong F$ or $M_{x,y}/\{x,y\} \cong F$. Then*

(i) *M has neither loops nor coloops,*
(ii) *if $M_{x,y}/\{x,y\} \cong F$ or $M_{x,y}/\{x\} \cong M^*(K_5)$, then M has at most one 2-circuit.*

Proof. The proof follows from Lemmas 2.1, 2.2 and the fact that F does not contain loops, coloops and 2-circuits. ■

Lemma 2.9 [10]**.** *A graph is minimal with respect to the matroid F_7 or F_7^* if and only if it is isomorphic to one of the three graphs G_1, G_2 and G_3 in Figure* 3.

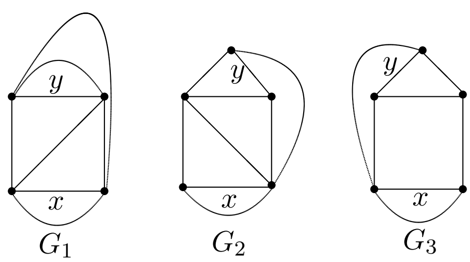


Figure 3

Lemma 2.10 [10]**.** *A graph is minimal with respect to the matroid $M^*(K_{3,3})$ if and only if it is isomorphic to one of the four graphs G_4, G_5, G_6 and G_7 presented in Figure* 4.

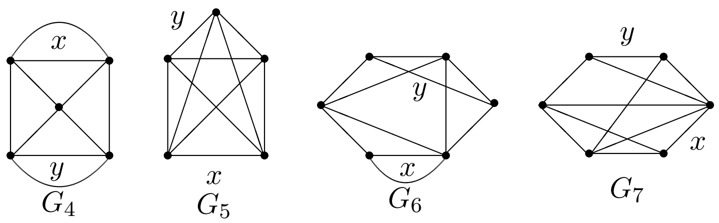


Figure 4

Lemma 2.11 [10]. *A graph is minimal with respect to the matroid $M^*(K_5)$ if and only if it is isomorphic to G_8 and G_9 presented in Figure* 5.

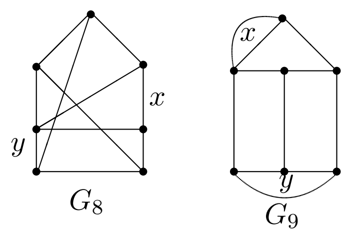


Figure 5

Lemma 2.12. *Let M be a cographic matroid. Then M is minimal with respect to the matroid F_7 or F_7^* if and only if M is isomorphic to one of the cycle matroids $M(G_1)$, $M(G_2)$ and $M(G_3)$, where G_1, G_2 and G_3 are the graphs in Figure* 6.

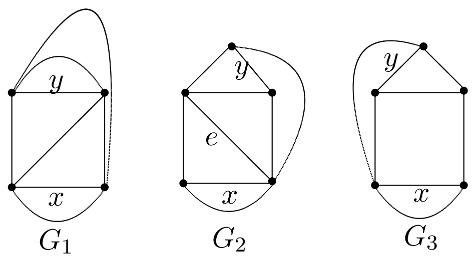


Figure 6

Proof. From the matrix representation it follows that $M(G_1)_{x,y}/\{x\} \cong F_7$, $M(G_2)_{x,y}/\{x, y\} \cong F_7$ and $M(G_3)_{x,y}/\{x\} \cong F_7^*$. Therefore, $M(G_1), M(G_2)$ and $M(G_3)$ are minimal with respect to F_7 or F_7^*.

Conversely, suppose M is minimal with respect to F_7 or F_7^*. Then there exist elements x, y such that $M_{x,y}/\{x\} \cong F_7$, or $M_{x,y}/\{x, y\} \cong F_7$, or $M_{x,y}/\{x\} \cong F_7^*$ or $M_{x,y}/\{x, y\} \cong F_7^*$. Suppose x and y are in series. Then, by Lemma 2.2(i), $M = M_{x,y}$. Therefore, M has F_7 or F_7^* as a minor, which is a contradiction to Theorem 2.4. Hence x and y are not in series in M. Thus, no two elements of M are in series in M. Now, the proof follows from Lemma 2.9. ■

Lemma 2.13. *Let M be a cographic matroid. Then M is minimal with respect to the matroid $M^*(K_{3,3})$ or $M^*(K_5)$ if and only if M is isomorphic to one of $M(G_i)$ for $i = 5, 6, 7, 12$ and to one of $M^*(G_j)$ for $j = 4, 8, 9, 10, 11, 13, 14, 15$, where the graphs G_i's and G_j's are shown in Figure 7.*

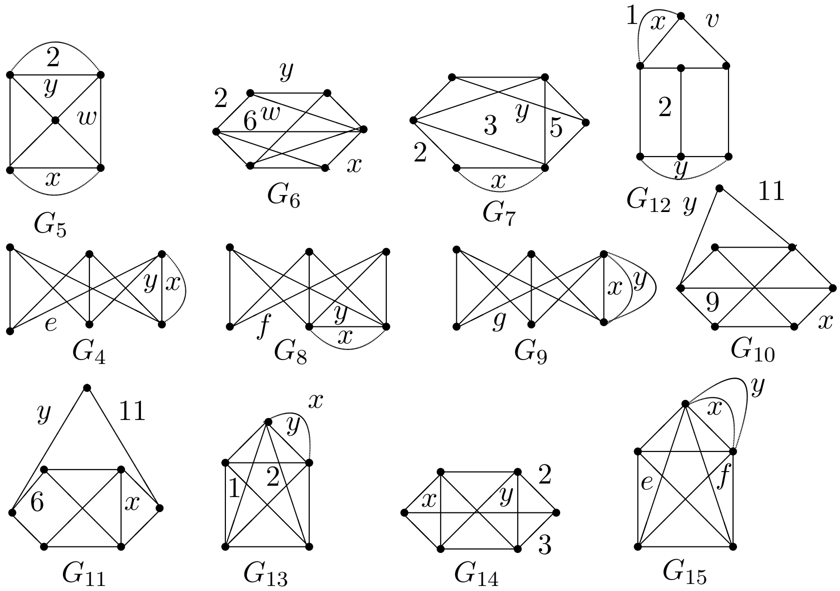


Figure 7

Proof. From the matrix representation, it follows that

$M^*(G_4)_{x,y}/\{x\} \cong M^*(K_{3,3})$, $M(G_5)_{x,y}/\{x\} \cong M^*(K_{3,3})$,
$M(G_6)_{x,y}/\{x,y\} \cong M^*(K_{3,3})$, $M(G_7)_{x,y}/\{x,y\} \cong M^*(K_{3,3})$,
$M^*(G_8)_{x,y}/\{x,y\} \cong M^*(K_{3,3})$, $M^*(G_9)_{x,y}/\{x,y\} \cong M^*(K_{3,3})$,
$M^*(G_{10})_{x,y}/\{x,y\} \cong M^*(K_{3,3})$, $M^*(G_{11})_{x,y}/\{x,y\} \cong M^*(K_{3,3})$,
$M(G_{12})_{x,y}/\{x\} \cong M^*(K_5)$, $M^*(G_{13})_{x,y}/\{x\} \cong M^*(K_5)$,
$M^*(G_{14})_{x,y}/\{x\} \cong M^*(K_5)$ and $M^*(G_{15})_{x,y}/\{x,y\} \cong M^*(K_5)$.

Therefore, $M(G_i)$ for $i = 5, 6, 7, 12$ and $M^*(G_j)$ for $j = 4, 8, 9, 10, 11, 13, 14, 15$ are minimal with respect to the matroid $M^*(K_{3,3})$ or $M^*(K_5)$.

Conversely, suppose that M is a minimal matroid with respect to the matroid $M^*(K_{3,3})$ or $M^*(K_5)$. Then there exist elements x and y of M such that $M_{x,y}/\{x\} \cong M^*(K_{3,3})$ or $M_{x,y}/\{x,y\} \cong M^*(K_{3,3})$ or $M_{x,y}/\{x\} \cong M^*(K_5)$ or $M_{x,y}/\{x,y\} \cong M^*(K_5)$.

Suppose x and y are in series in M. Then, by Lemma 2.2(i), $M = M_{x,y}$. Hence $M/\{x\} \cong M^*(K_{3,3})$ or $M/\{x,y\} \cong M^*(K_{3,3})$ or $M/\{x\} \cong M^*(K_5)$ or $M/\{x,y\} \cong M^*(K_5)$; i.e. $M^* \setminus \{x\} \cong M(K_{3,3})$ or $M^* \setminus \{x,y\} \cong M(K_{3,3})$ or $M^* \setminus \{x\} \cong M(K_5)$ or $M^* \setminus \{x,y\} \cong M(K_5)$. Since x and y are in parallel in M^*, it follows that $M \cong M^*(G_i)$ for $i = 4, 8, 9, 13, 15$.

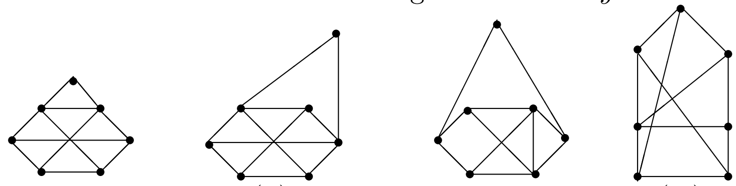
Now, suppose x and y are not in series in M. Then $M \neq M_{x,y}$. By Lemma 2.2(ii), $r(M_{x,y}) = r(M) + 1$.

Case (i). $M_{x,y}/\{x\} \cong M^*(K_{3,3})$. We claim that M is graphic. By Theorems 2.3 and 2.4, it suffices to prove that M does not have any of the matroids F_7, F_7^*, $M^*(K_{3,3})$ and $M^*(K_5)$ as a minor. As M is cographic, F_7 and F_7^* are excluded minors for M. Further, $|E(M)| = 10$ and, by Lemma 2.2 (ii), $r(M) = r(M_{x,y}) - 1 = r(M_{x,y}/\{x\}) = r(M^*(K_{3,3})) = 4$. Hence M cannot have a minor isomorphic to $M^*(K_5)$. Assume that M has a minor isomorphic to $M^*(K_{3,3})$. There exists an element q in M such that $M \setminus q \cong M^*(K_{3,3})$. Therefore $M^*/q \cong M(K_{3,3})$. Since $M^*(K_{3,3})$ is Eulerian, by Lemma 2.2(v), M is Eulerian and hence M^* is bipartite. By Lemm 2.8(i), q is neither a loop nor a coloop. Hence there exists a circuit C in M^* containing q. Since C is an even circuit, C/q is an odd circuit in $M^*/q \cong M(K_{3,3})$, a contradiction. Thus M is graphic. Hence $M \cong M(G)$, where G is a planar graph. It follows from the proof of Lemma 2.10 that $M \cong M(G_5)$ of Figure 7.

Case (ii). $M_{x,y}/\{x, y\} \cong M^*(K_{3,3})$. If M is graphic, then by Lemma 2.10, $M \cong M(G_6)$ or $M(G_7)$ of Figure 7. Suppose that M is not graphic. As M is cographic, $M \cong M^*(G)$ for some graph G. Further, G has 7 vertices and 11 edges because $r(M^*) = 6$. As $|E(M^*(K_{3,3}))| = 9$, $r(M^*(K_{3,3})) = 4$, $M \setminus \{p\}/\{q\} \cong M^*(K_{3,3})$ for some elements p, q of M. Therefore $M^*/\{p\} \setminus \{q\} \cong M(K_{3,3})$. Since M has no 2-cocircuit, G is simple. Further, G is non-planar. By Lemma 2.8(ii), M has at most one 2-circuit and hence G has at most one vertex of degree 2. Therefore, the degree sequence of G is (4,3,3,3,3,3,3), (4,4,3,3,3,3,2) or (5,3,3,3,3,3,2).

Consider the degree sequence (5,3,3,3,3,3,2). A non-planar simple graph with degree sequence (5,3,3,3,3,3,2) can be obtained from a non-planar simple graph with degree sequence (4,3,3,3,3,2) or (5,3,3,3,2,2) by adding a vertex of degree 2. But there is no non-planar simple graph with any of these two degree sequences see [4]. So, we discard the degree sequence (5,3,3,3,3,3,2).

Since all cocircuits of $M^*(K_{3,3})$ are even and M has no odd cocircuit, the graph G cannot have an i-circuit containing both x and y for $i = 3, 4, 5, 7$.



(i) (ii) (iii) (iv)

Figure 8

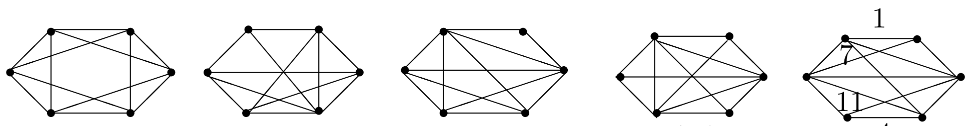
Now, consider the degree sequence (4,3,3,3,3,3,3). By [10], there is only one non-planar simple graph of degree sequence (4,3,3,3,3,3,3), as shown in Figure 8(iv).

In this graph every pair of edges is contained in an i-circuit, for some $i = 3, 4, 5, 7$. Hence we discard this graph.

A non-planar simple graph with degree sequence (4,4,3,3,3,3,2) can be obtained from a non-planar simple graph with degree sequence (3,3,3,3,3,3) or (4,4,3,3,2,2) by adding a vertex of degree 2. It follows from [4] that every non-planar simple graph with degree sequence (4,4,3,3,3,3,2) is isomorphic to one of the first three graphs of Figure 8. Graph (i) is discarded because every pair of edges is contained in an i-circuit for some $i = 3, 4, 5, 7$. The remaining two graphs are nothing but the graphs G_{10} and G_{11} in the statement of the lemma.

Case (iii). $M_{x,y}/\{x\} \cong M^*(K_5)$. If M is graphic, then, by Lemma 2.11, we get two graphs which one of them is a graph (iv) of Figure 8, which is already discarded. So, $M \cong M(G_{12})$ of Figure 7. Suppose that M is not graphic. As M is cographic, $M = M^*(G)$ for some non-planar graph G. Further, G has 6 vertices and 11 edges because $r(M^*) = 5$. By Lemma 2.8(i), M has no loops and coloops and also no two elements of M are in series, G is simple and has minimum degree at least 2. Also, by Lemma 2.8(ii), G has at most one vertex of degree 2. Hence the degree sequence of G is (4,4,4,4,3,3) or (4,4,4,4,4,2). By [4], the graph G_{14} of Figure 7 is the only one non-planar simple graph with the degree sequence (4,4,4,4,3,3). Also, there is only one non-planar simple graph with degree sequence (4,4,4,4,4,2) see [4]. In this graph, any pair of edges are either in a 3-circuit or a 4-circuit. If G is isomorphic to this graph, then x, y belong to a 3-cocircuit or a 4-cocircuit $C^*$of M and hence $C^* - \{x, y\}$ is a 1-cocircuit or a 2-cocircuit in $M_{x,y}/\{x\}$, a contradiction.

Case (iv). $M_{x,y}/\{x, y\} \cong M^*(K_5)$. First we show that M is graphic. Suppose that M is not graphic. Then M has $M^*(K_5)$ or $M^*(K_{3,3})$ as a minor. On the contrary, suppose M has $M^*(K_5)$ or $M^*(K_{3,3})$ as a minor. As $r(M) = 7$ and $|E(M)| = 12$, $M\backslash\{p\}/\{q\} \cong M^*(K_5)$ for some elements $p, q \in E(M)$. This implies that $M^*/\{p\} \setminus \{q\} \cong M(K_5)$. Also, $M/\{n, m, s\} \cong M^*(K_{3,3})$ for some elements $n, m, s \in E(M)$. This implies that $M^* \setminus \{n, m, s\} \cong M(K_{3,3})$. Thus $M \cong M^*(G)$, where G is a non-planar simple graph with 6 vertices and 12 edges. By Lemma 2.8(ii), G has at most one vertex of degree 2. Therefore the degree sequence of G is (4,4,4,4,4,4), (5,4,4,4,4,3), (5,5,4,4,3,3), (5,5,5,3,3,3) or (5,5,4,4,4,2). By [4], there is only one non-planar simple graph for each of these sequences, as shown in Figure 9.



(i) (ii) (iii) (iv) (v)

Figure 9

It follows from the nature of cocircuits of $M^*(K_5)$, that both x, y do not belong to an i-circuit for $i = 3, 4$ nor to a j-cocircuit for $j = 3, 4, 5, 7$. These conditions are not satisfied by any pair of edges of the first 4 graphs of Figure 9. Hence we discard these graphs. Further, in the graph (v) of Figure 9 each pair of edges belongs to an i-circuit for $i = 3, 4$ and to a j-cocircuit for $j = 3, 4, 5, 7$, except the pairs (1,4), (1,11) and (4,7). For these pairs, there is a 5-circuit in $M_{x,y}/\{x, y\}$ and hence it cannot be isomorphic to $M^*(K_5)$ since $M^*(K_5)$ has 5 circuits of size 4 and 10 circuits of size 6. Thus G cannot be obtained from this graph. So M does not have $M^*(K_5)$ or $M^*(K_{3,3})$ as a minor. We conclude that M is graphic. Now the proof follows from Lemma 2.11 ■

Now, we use Lemmas 2.12 and 2.13 to prove Theorem 1.3.

***Proof of Theorem* 1.3.** Let M be a cographic matroid. On combining Corollary 2.7 and Lemmas 2.12 and 2.13, it follows that $M_{x,y}$ is graphic for every pair $\{x, y\}$ of elements of M if and only if M has no minor isomorphic to any of the matroids $M(G_i)$, $i = 1, 2, 3, 5, 6, 7, 12$ and $M^*(G_j)$, $j = 4, 8, 9, 10, 11, 13, 14, 15$ where the graphs G_i and G_j are shown in the statements of the Lemmas 2.12 and 2.13. However, we have $M(G_3) \cong M(G_2) \setminus \{e\} \cong M(G_5) \setminus \{2, w\} \cong M(G_6)/\{2\} \setminus \{6, w\} \cong M(G_7)/\{2\} \setminus \{3, 5\} \cong M(G_{12}) \setminus \{1\}/\{v, 2\}$; $M^*(G_1) \cong M(G_4)\setminus\{x, e\} \cong M(G_8)\setminus\{x, y, f\} \cong M(G_9)\setminus\{x, y, g\} \cong M(G_{10})/\{11\}\setminus\{9, y\}$ and $M^*(G_3) \cong M(G_{11})/\{6, y\} \setminus \{11\} \cong M(G_{13}) \setminus \{1, 2, x\} \cong M(G_{14})/\{y\} \setminus \{2, 3\} \cong M(G_{15}) \setminus \{e, f, x, y\}$.

This means that
$M(G_1) \cong M^*(G_4)/\{x, e\} \cong M^*(G_8)/\{x, y, f\} \cong M^*(G_9)/\{x, y, g\}$
$\cong M^*(G_{10})/\{9, y\} \setminus \{11\}$ and $M(G_3) \cong M^*(G_{11})/\{11\} \setminus \{6, y\}$
$\cong M^*(G_{13})/\{1, 2, x\} \cong M^*(G_{14})/\{2, 3\} \setminus \{y\} \cong M^*(G_{15})/\{e, f, x, y\}$.

Thus, $M_{x,y}$ is graphic if and only if M has no minor isomorphic to any of the matroids $M(G_i)$ for $i = 1, 3$. But the graphs G_i are precisely the graphs given in the statement of the theorem. This completes the proof. ■

Acknowledgements

I am thankful to the referees for recommending the present revised version of the rather elaborate original version of this paper.

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Received 4 March 2013
Revised 7 February 2014
Accepted 7 February 2014