Problems Column

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CLIQUE PARTS INDEPENDENT OF REMAINDERS

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Let p and t stand for positive integers. Let R denote an edge subset of size $|R| = \binom{p}{2} \bmod t$ in the complete graph K_p. Call R a *remainder* (or an *edge t-remainder*) in the clique K_p.

Conjecture L (L reminds of floor symbol)**.** The floor class $\lfloor K_p/t \rfloor$ is nonempty. In other words, there exists a graph F such that, for each edge t-remainder R in K_p, F is a tth part of $K_p - R$, i.e., $F \in \lfloor K_p/t \rfloor$.

Conjecture L implies the following conjecture stated in [2].

Conjecture L*. For each edge t-remainder R in K_p, there is an $F_R \in (K_p - R)/t =: \lfloor K_p/t \rfloor_R$.

Theorem L′ (Skupień [2])**.** *There exists an edge t-remainder R in K_p such that the floor class $\lfloor K_p/t \rfloor_R$ is nonempty.*

Plantholt's theorem [1] on chromatic index is equivalent to the truth of Conjecture L with $t = p - 1$ and p being odd.

Conjecture L can be seen true for many pairs p, t, e.g., if $t \geq p - 1$ or t is small: $t \leq 5$. If t is a constant ($t \geq 4$), both Conjectures can be reduced to some values of p in the interval $t + 2 \leq p \leq 4t - 5$.

References

[1] M. Plantholt, *The chromatic index of graphs with a spanning star*, J. Graph Theory **5** (1981) 45–53.

[2] Z. Skupień, *The complete graph t-packings and t-coverings*, Graphs Combin. **9** (1993) 353–363.

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