

2-DISTANCE COLORINGS OF INTEGER DISTANCE GRAPHS

BRAHIM BENMEDJDOUB, ISMA BOUCHEMAKH

Faculty of Mathematics, Laboratory L'IFORCE
University of Sciences and Technology Houari Boumediene (USTHB)
B.P. 32 El-Alia, Bab-Ezzouar, 16111 Algiers, Algeria

e-mail: brahimro@hotmail.com
isma_bouchemakh2001@yahoo.fr

AND

ÉRIC SOPENA

Univ. Bordeaux, Bordeaux INP, CNRS, LaBRI, UMR5800
F-33400 Talence, France

e-mail: Eric.Sopena@labri.fr

Abstract

A 2-distance k -coloring of a graph G is a mapping from $V(G)$ to the set of colors $\{1, \dots, k\}$ such that every two vertices at distance at most 2 receive distinct colors. The 2-distance chromatic number $\chi_2(G)$ of G is then the smallest k for which G admits a 2-distance k -coloring. For any finite set of positive integers $D = \{d_1, \dots, d_\ell\}$, the integer distance graph $G = G(D)$ is the infinite graph defined by $V(G) = \mathbb{Z}$ and $uv \in E(G)$ if and only if $|v - u| \in D$. We study the 2-distance chromatic number of integer distance graphs for several types of sets D . In each case, we provide exact values or upper bounds on this parameter and characterize those graphs $G(D)$ with $\chi_2(G(D)) = \Delta(G(D)) + 1$.

Keywords: 2-distance coloring, integer distance graph.

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