Discussiones Mathematicae
Graph Theory 36 (2016) 857–865
doi:10.7151/dmgt.1892

EDGE-TRANSITIVE LEXICOGRAPHIC AND CARTESIAN PRODUCTS

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Abstract

In this note connected, edge-transitive lexicographic and Cartesian products are characterized. For the lexicographic product $G \circ H$ of a connected graph G that is not complete by a graph H, we show that it is edge-transitive if and only if G is edge-transitive and H is edgeless. If the first factor of $G \circ H$ is non-trivial and complete, then $G \circ H$ is edge-transitive if and only if H is the lexicographic product of a complete graph by an edgeless graph. This fixes an error of Li, Wang, Xu, and Zhao [11]. For the Cartesian product it is shown that every connected Cartesian product of at least two non-trivial factors is edge-transitive if and only if it is the Cartesian power of a connected, edge- and vertex-transitive graph.

Keywords: edge-transitive graph, vertex-transitive graph, lexicographic product of graphs, Cartesian product of graphs.

2010 Mathematics Subject Classification: 05C76, 05C25.

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Received 16 April 2015

Revised 29 December 2015

Accepted 29 December 2015