

## SHARP UPPER BOUNDS FOR GENERALIZED EDGE-CONNECTIVITY OF PRODUCT GRAPHS

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### Abstract

The generalized  $k$ -connectivity  $\kappa_k(G)$  of a graph  $G$  was introduced by Hager in 1985. As a natural counterpart of this concept, Li *et al.* in 2011 introduced the concept of generalized  $k$ -edge-connectivity which is defined as  $\lambda_k(G) = \min\{\lambda(S) : S \subseteq V(G) \text{ and } |S| = k\}$ , where  $\lambda(S)$  denote the maximum number  $\ell$  of pairwise edge-disjoint trees  $T_1, T_2, \dots, T_\ell$  in  $G$  such that  $S \subseteq V(T_i)$  for  $1 \leq i \leq \ell$ . In this paper, we study the generalized edge-connectivity of product graphs and obtain sharp upper bounds for the generalized 3-edge-connectivity of Cartesian product graphs and strong product graphs. Among our results, some special cases are also discussed.

**Keywords:** generalized edge-connectivity, Cartesian product, strong product, lexicographic product.

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