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INTEGRAL CAYLEY SUM GRAPHS AND GROUPS

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Abstract

For any positive integer k, let $\mathcal{A}_k$ denote the set of finite abelian groups G such that for any subgroup H of G all Cayley sum graphs $\mathrm{CayS}(H, S)$ are integral if $|S| = k$. A finite abelian group G is called Cayley sum integral if for any subgroup H of G all Cayley sum graphs on H are integral. In this paper, the classes $\mathcal{A}_2$ and $\mathcal{A}_3$ are classified. As an application, we determine all finite Cayley sum integral groups.

Keywords: Cayley sum graph, integral graph, Cayley sum integral group.

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